CS 220: Recursion

The Art of Self Reference

Meena Syamkumar
Michael Doescher

Part 2 of 3 - Data Structures

- Lists and Dictionaries

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- CSV and JSON

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- Lists and Dictionaries
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- Objects and References
- Fancy Functions
  - Recursion
  - Generators
  - Functions are Objects

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- Fancy Functions
  - Recursion
  - Generators
  - Functions are Objects
- Files
- Errors

Goal: use self-reference is a meaningful way

***Hofstadter's Law**: "It always takes longer than you expect, even when you take into account **Hofstadter's Law**."*

(From Gödel, Escher, Bach)

good advice for CS assignments!

"Dialectical Materialism is materialism that involves dialectic."

https://en.wikipedia.org/wiki/Circular_definition

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"Dialectical Materialism is materialism that involves dialectic."

"The Marxist theory (adopted as the official philosophy of the Soviet communists) that political and historical events result from the conflict of social forces and are interpretable as a series of contradictions and their solutions. The conflict is believed to be caused by material needs."

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***mountain**: "a landmass that projects conspicuously above its surroundings and is higher than a **hill**"*

***hill**: "a usually rounded natural elevation of land lower than a **mountain**"*

(Example of **unhelpful** self reference from Merriam-Webster dictionary)

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Overview: Learning Objectives

Recursive definitions and recursive information

- What is a **recursive definition/structure**?
- Arbitrarily vs. infinitely

Recursive code

- What is **recursive code**?
- Why write recursive code?
- Where do computers keep local variables for recursive calls?
- What happens to programs with **infinite recursion**?

Read *Think Python*

- Ch 5: “Recursion” through “Infinite Recursion”
- Ch 6: “More Recursion” through end

What is Recursion?

Recursive definitions

- Contain the term in the body
- Dictionaries, mathematical definitions, etc

A number **x** is a positive even number if:

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Recursive definitions

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A number **x** is a positive even number if:

- **x** is 2

  OR

- **x** equals another positive even number plus two

What is Recursion?

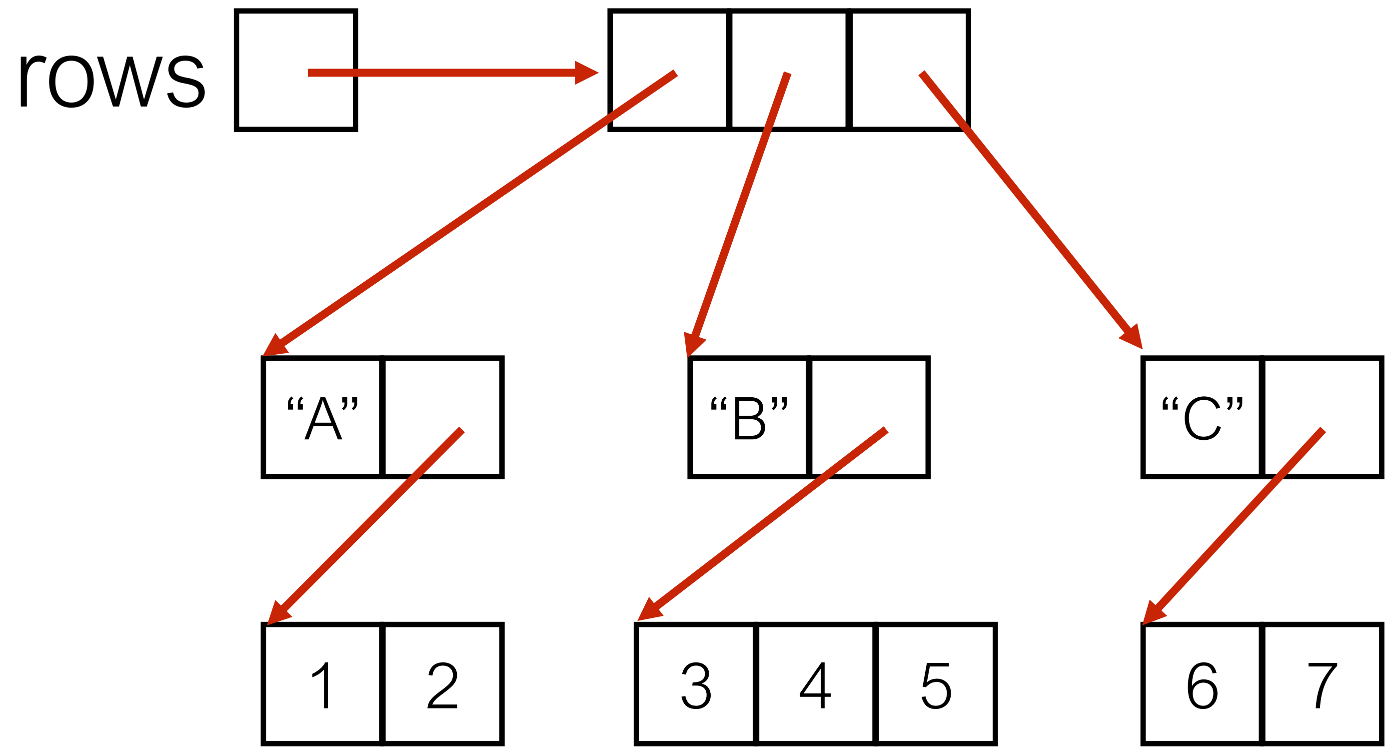
Recursive definitions

- Contain the term in the body
- Dictionaries, mathematical definitions, etc

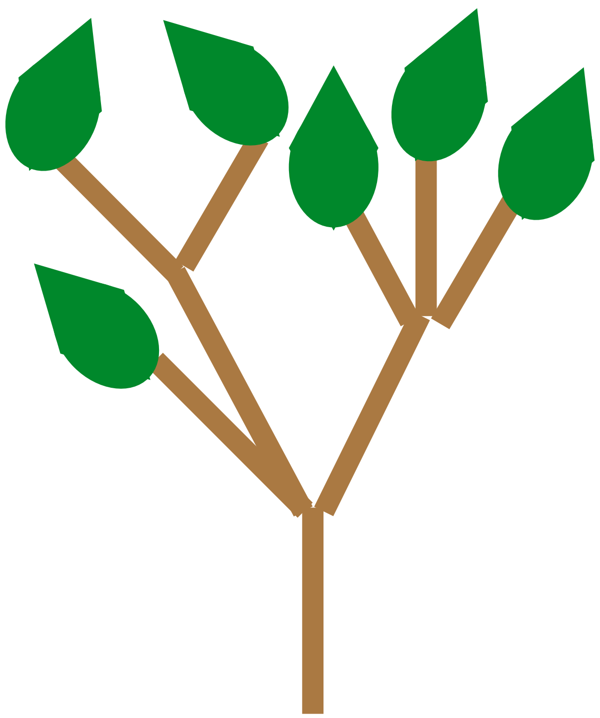
Recursive structures may refer to structures of the same type

- data structures or real-world structures

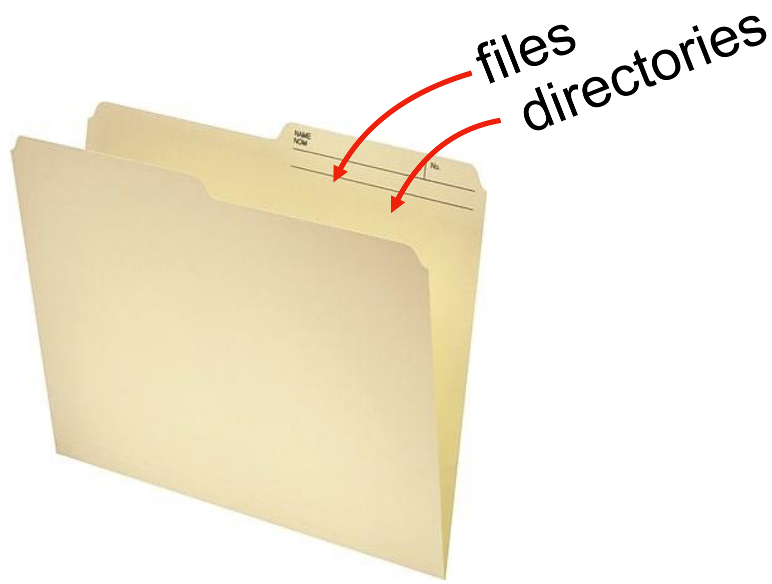
```
rows = [
   ["A",[1,2]],
   ["B",[3,4,5]],
   ["C",[6,7]]
]
```



Recursive structures are EVERYWHERE!



nature



files

```
{
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                "total":100},
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  }
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```

formats

Example: Trees (Finite Recursion)

Term: branch

Def: wooden stick, with an end splitting into other branches, OR terminating with a leaf

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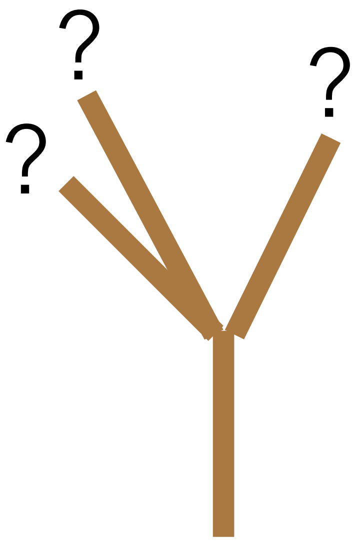

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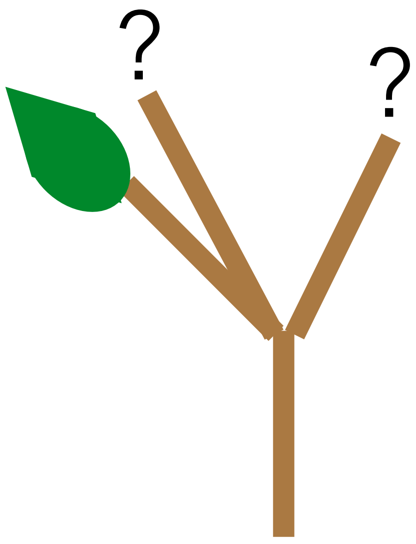

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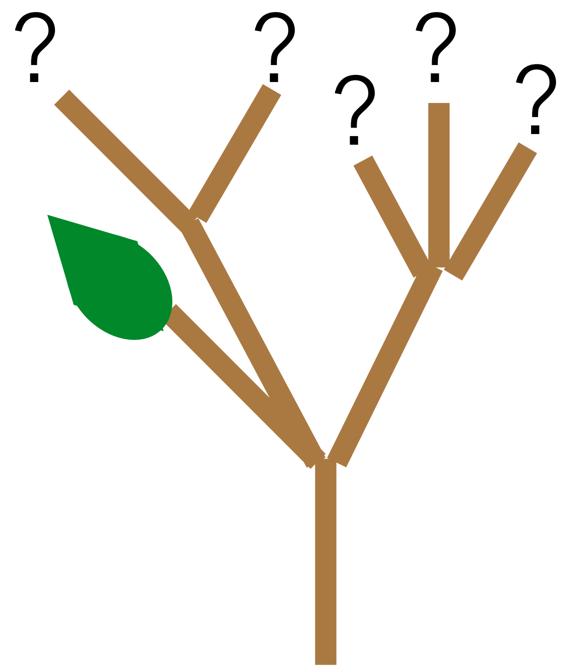

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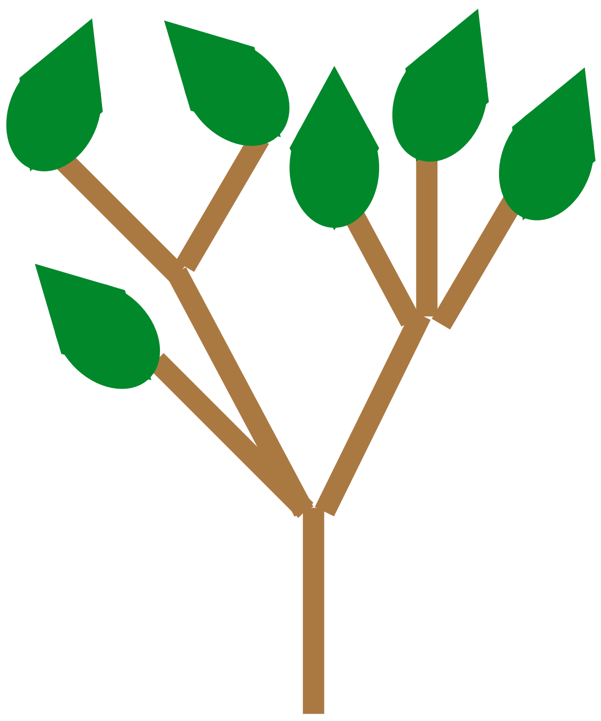

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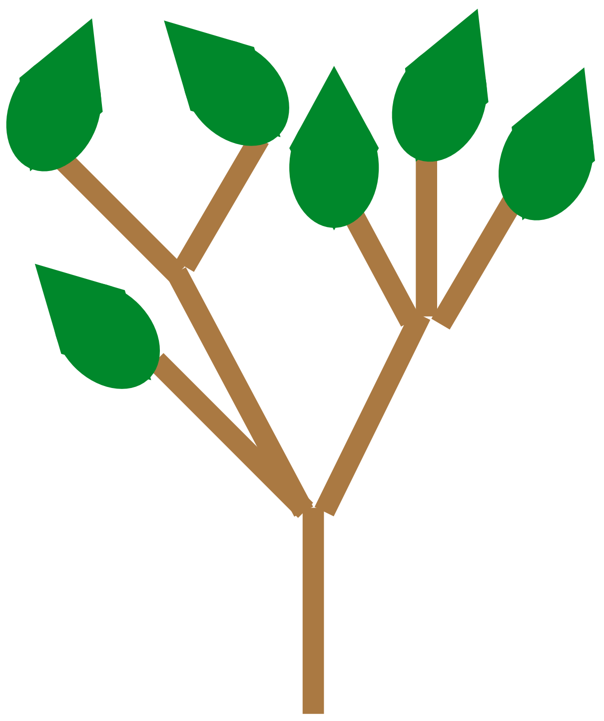

trees are arbitrarily large: **recursive case** allows indefinite growth

arbitrarily != infinitely

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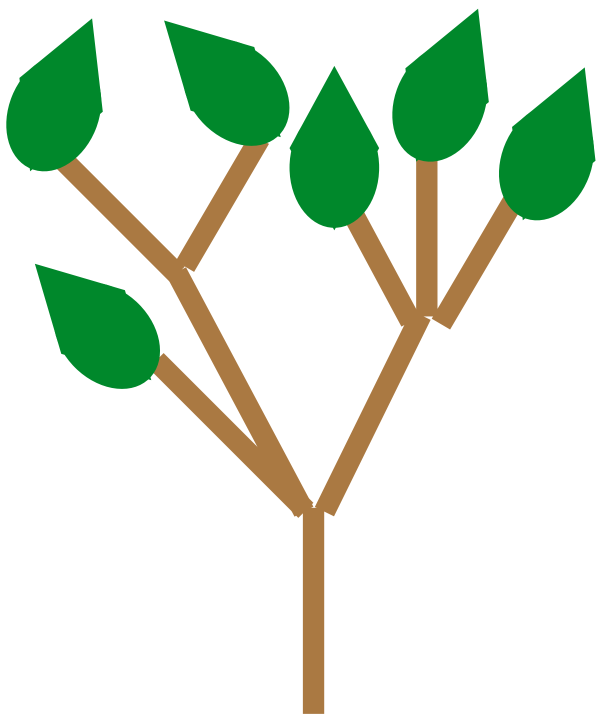

Def: wooden stick, with an end
splitting into other branches, OR
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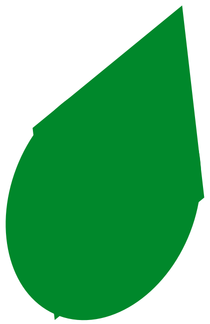
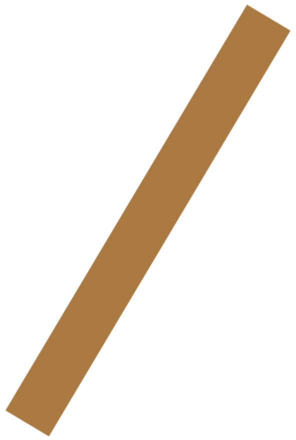
trees are finite:
eventual **base case**
allows completion

trees are arbitrarily large:
recursive case allows
indefinite growth

arbitrarily != infinitely



base case (leaf)



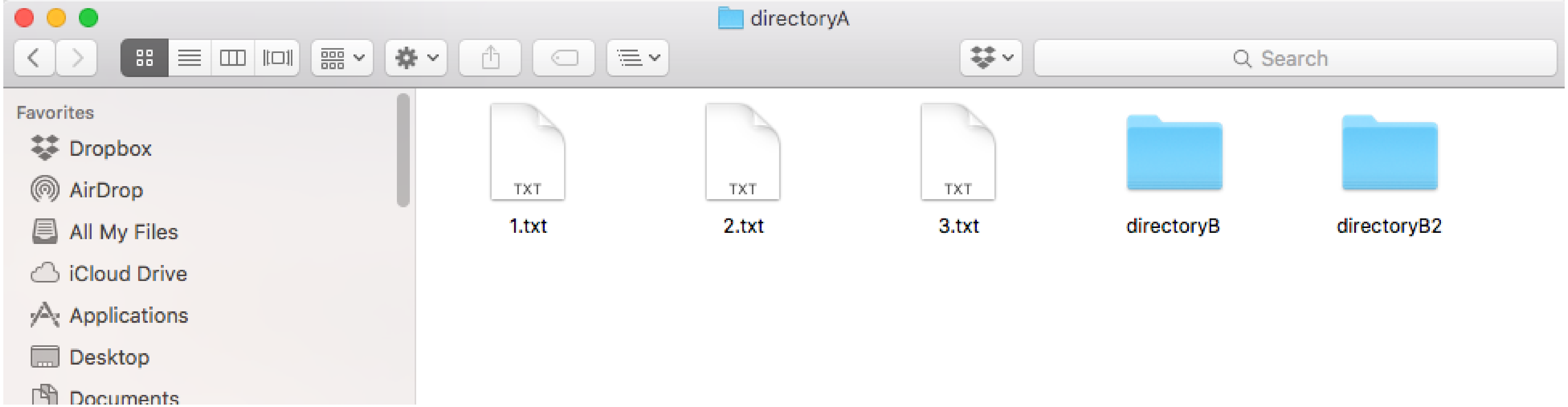
recursive case (branch)

Example: Directories (aka folders)

Term: directory

recursive because def contains term

Def: a collection of files and directories



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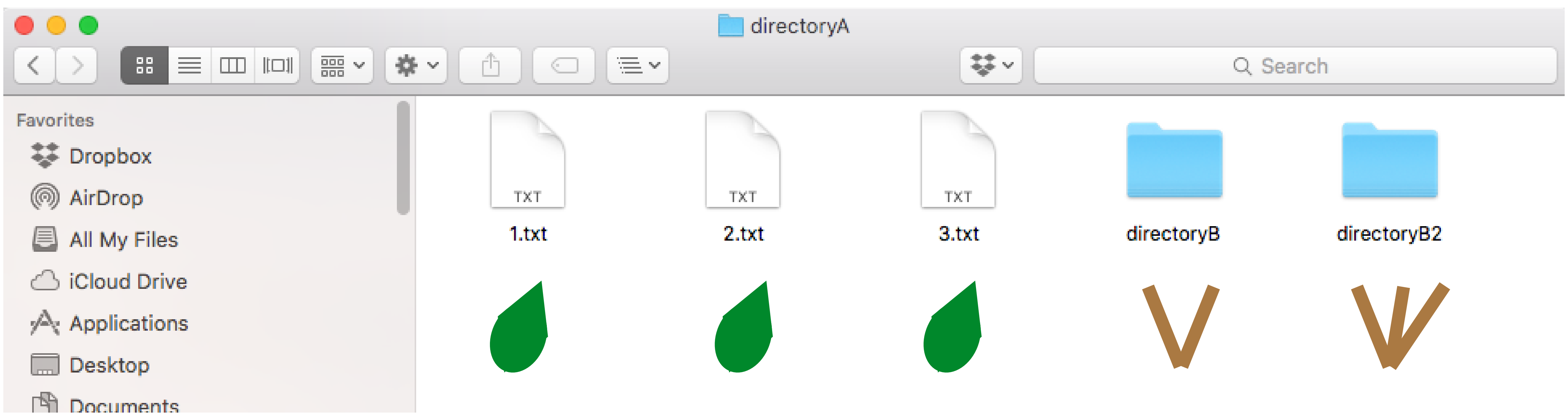

file system tree

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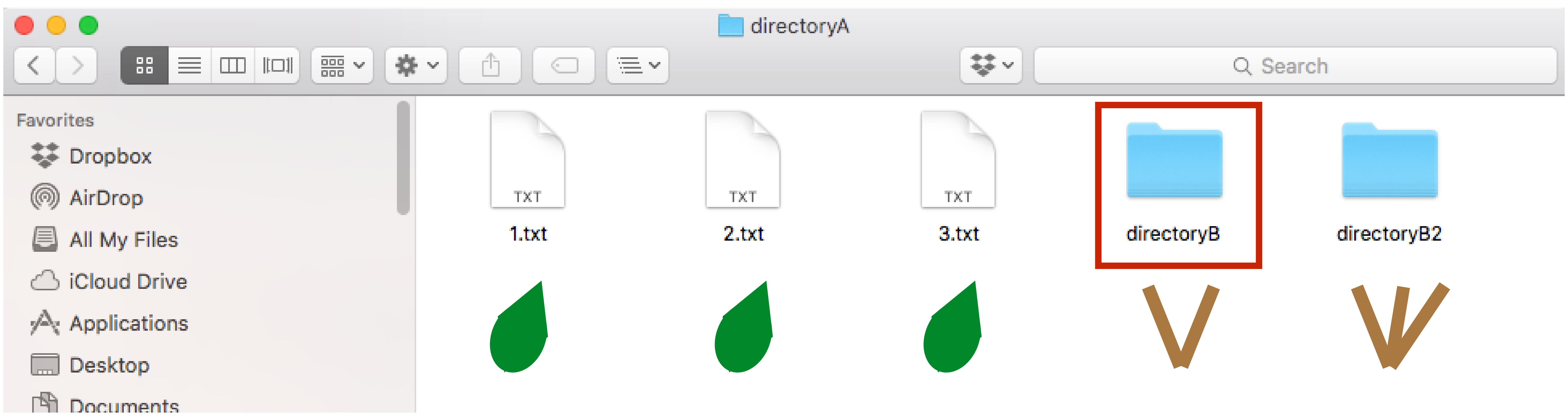

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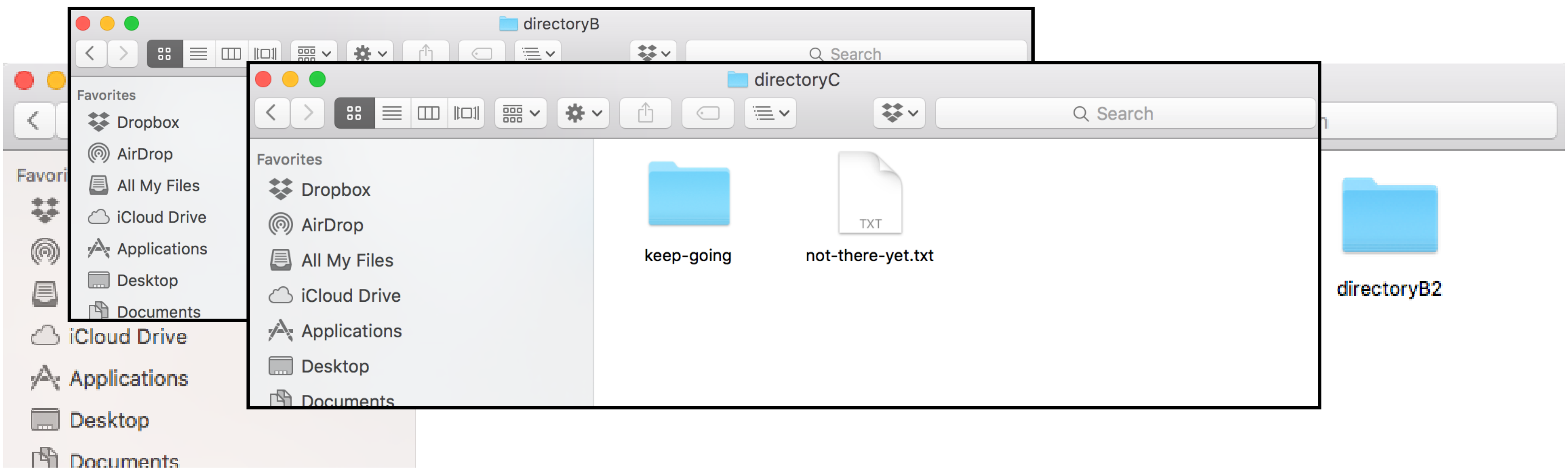

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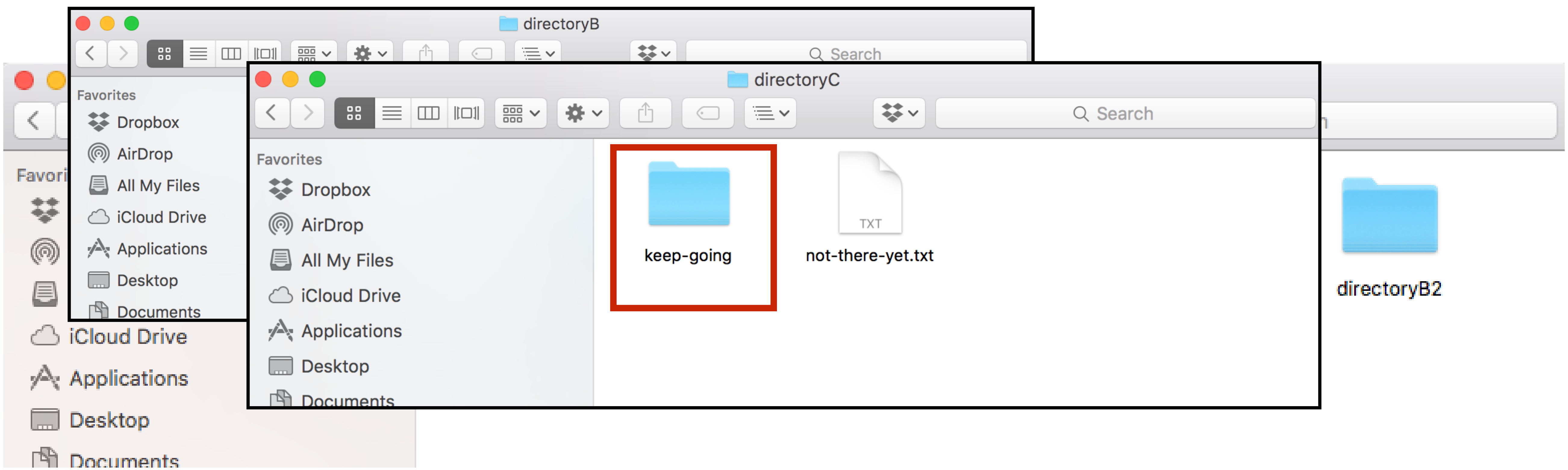

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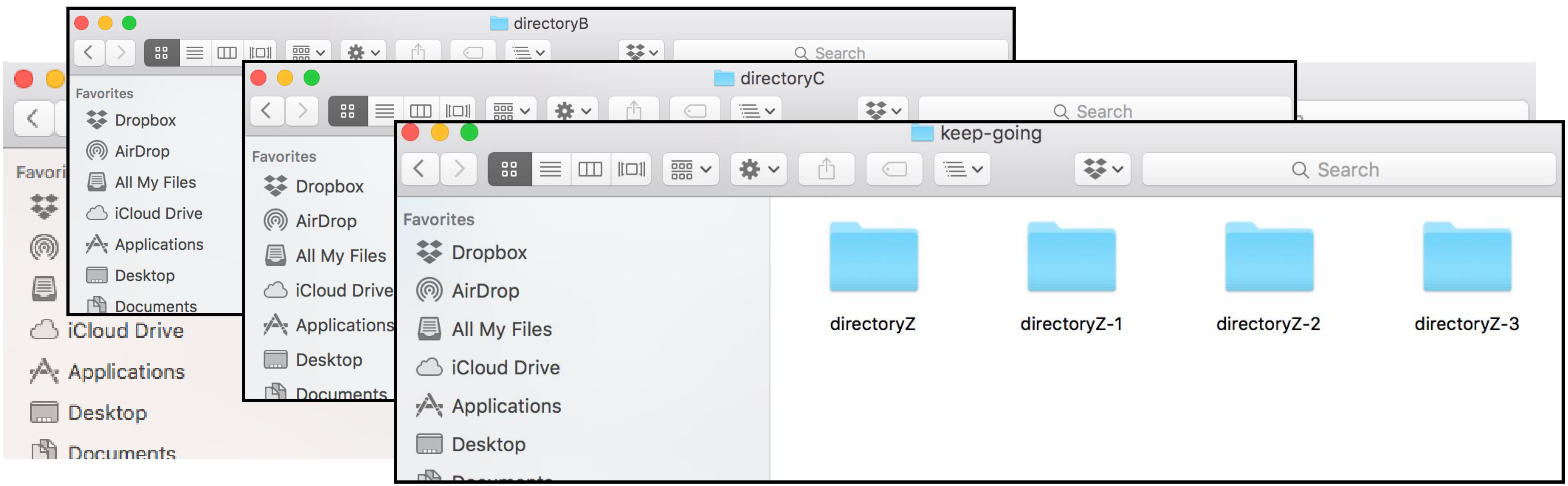

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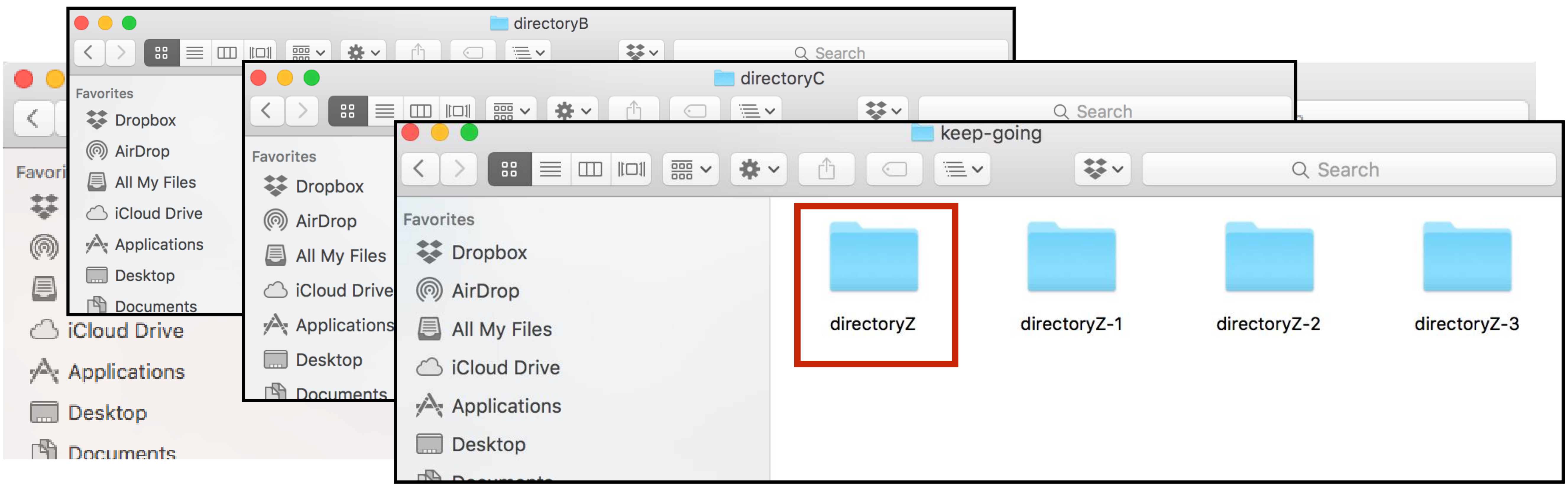

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Example: (simplified) JSON Format

Example JSON Dictionary:

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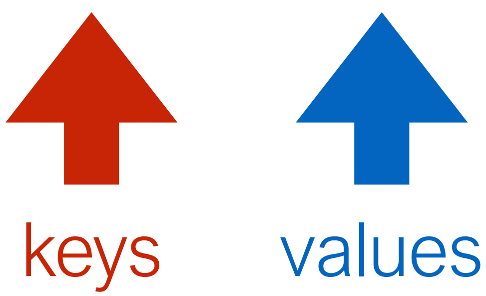

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recursive self reference isn't always direct!

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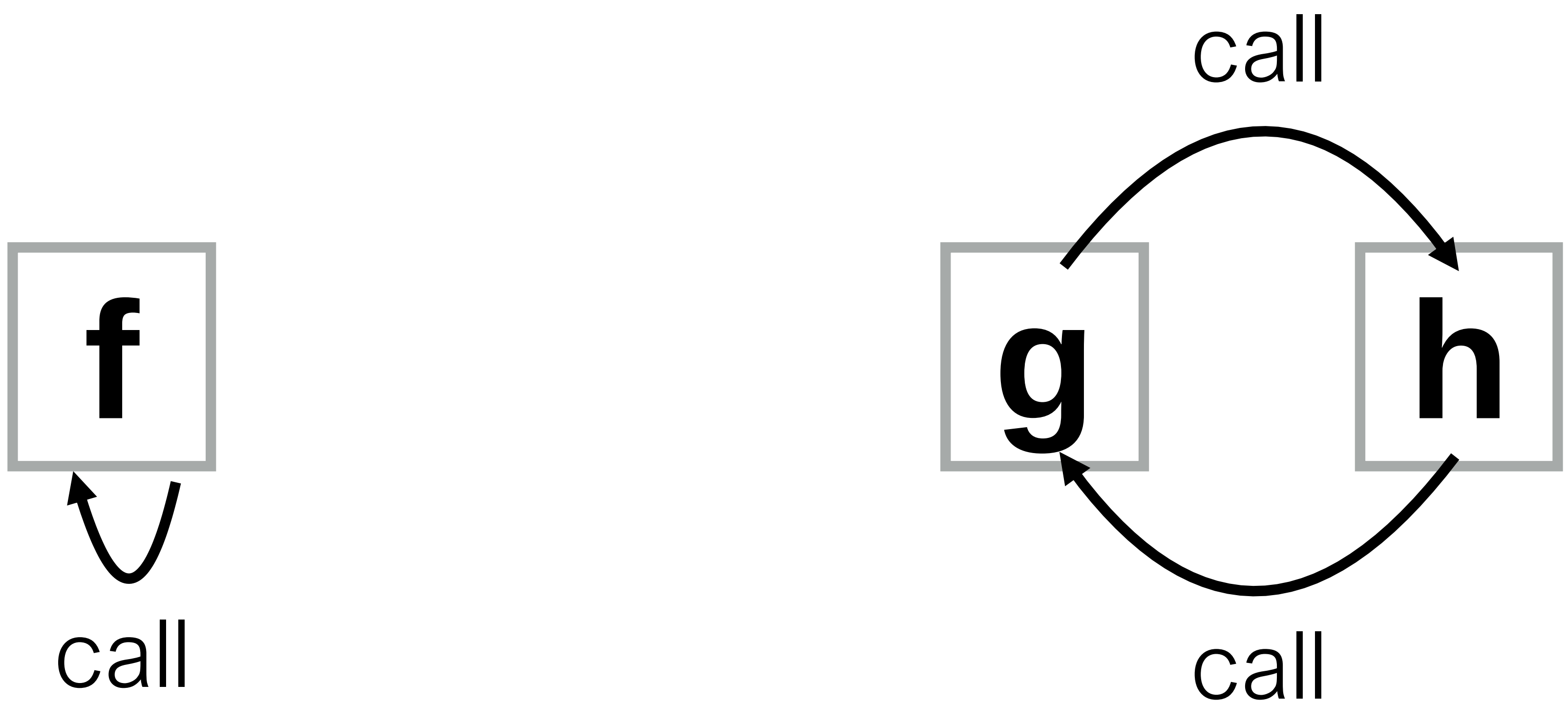
Recursive Code

What is it?

- A function that calls itself (possible indirectly)

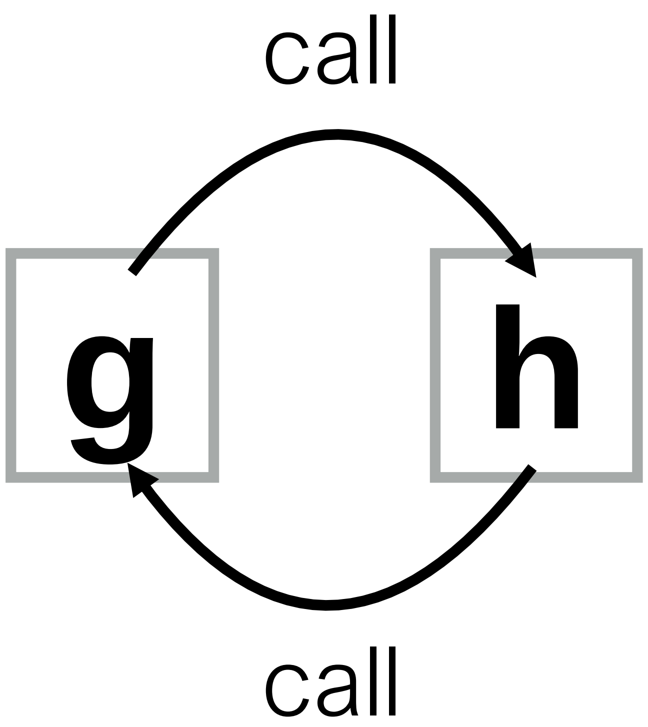


Recursive Code

What is it?

- A function that calls itself (possible indirectly)

```
def f():
    # other code
    f()
    # other code
```



Recursive Code

What is it?

- A function that calls itself (possible indirectly)

```
def f():
   # other code
   f()
   # other code
```

```
def g():
   # other code
   h()
   # other code

def h():
   # other code
   g()
   # other code
```

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

Motivation: don’t know how big the data is before execution

- Need either **iteration** or **recursion**
- In theory, these techniques are equally powerful

Recursive Code

What is it?

- A function that calls itself (possible indirectly)

Motivation: don't know how big the data is before execution

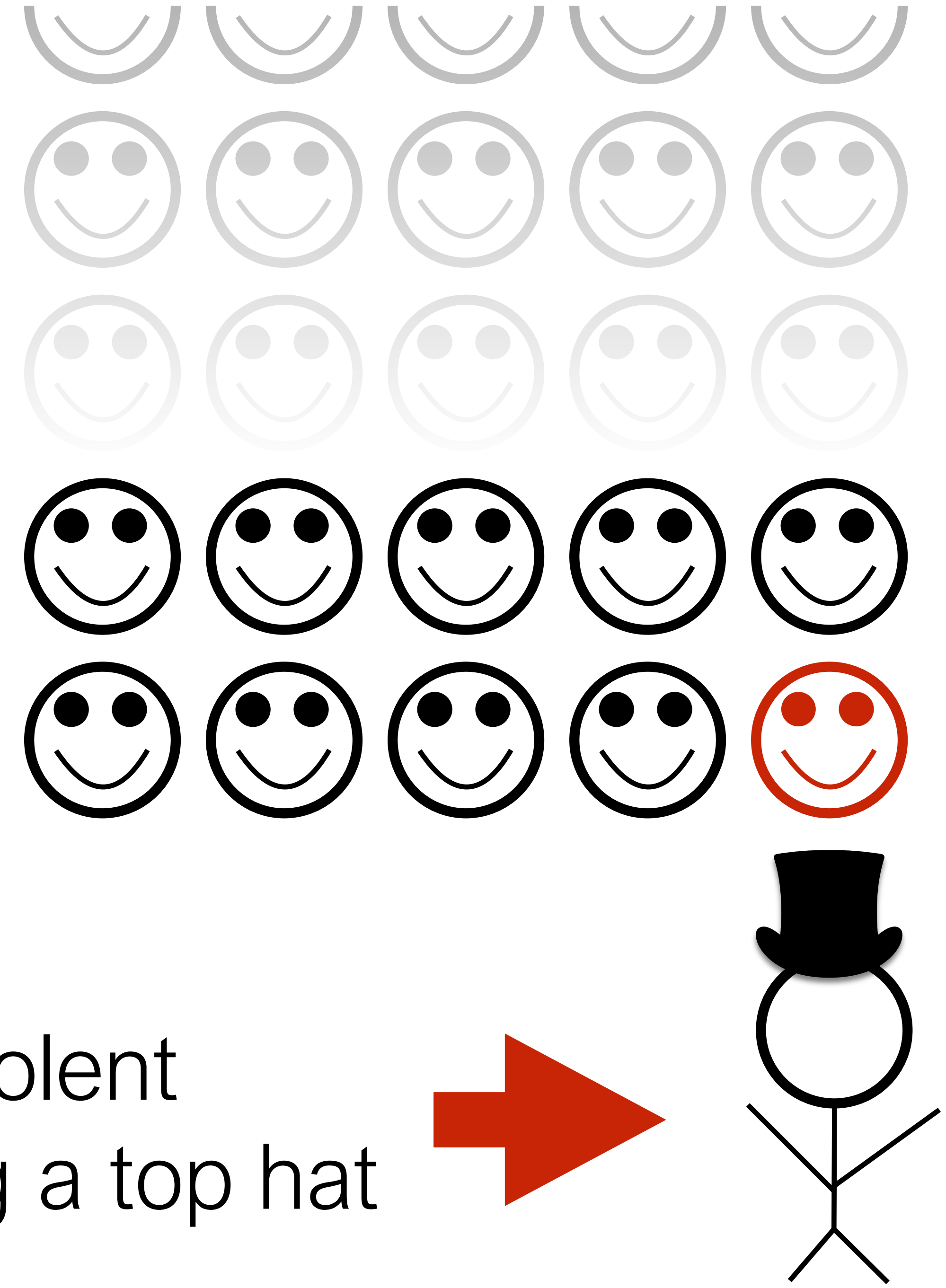
- Need either **iteration** or **recursion**
- In theory, these techniques are equally powerful

Why recurse? (instead of always iterating)

- in practice, often easier
- recursive code corresponds to recursive data
- reduce a big problem into a smaller problem

https://texastreesurgeons.com/services/tree-removal/

Recursive Student Counting



eager CS 220 students
in the front row

wise and benevolent
teacher wearing a top hat

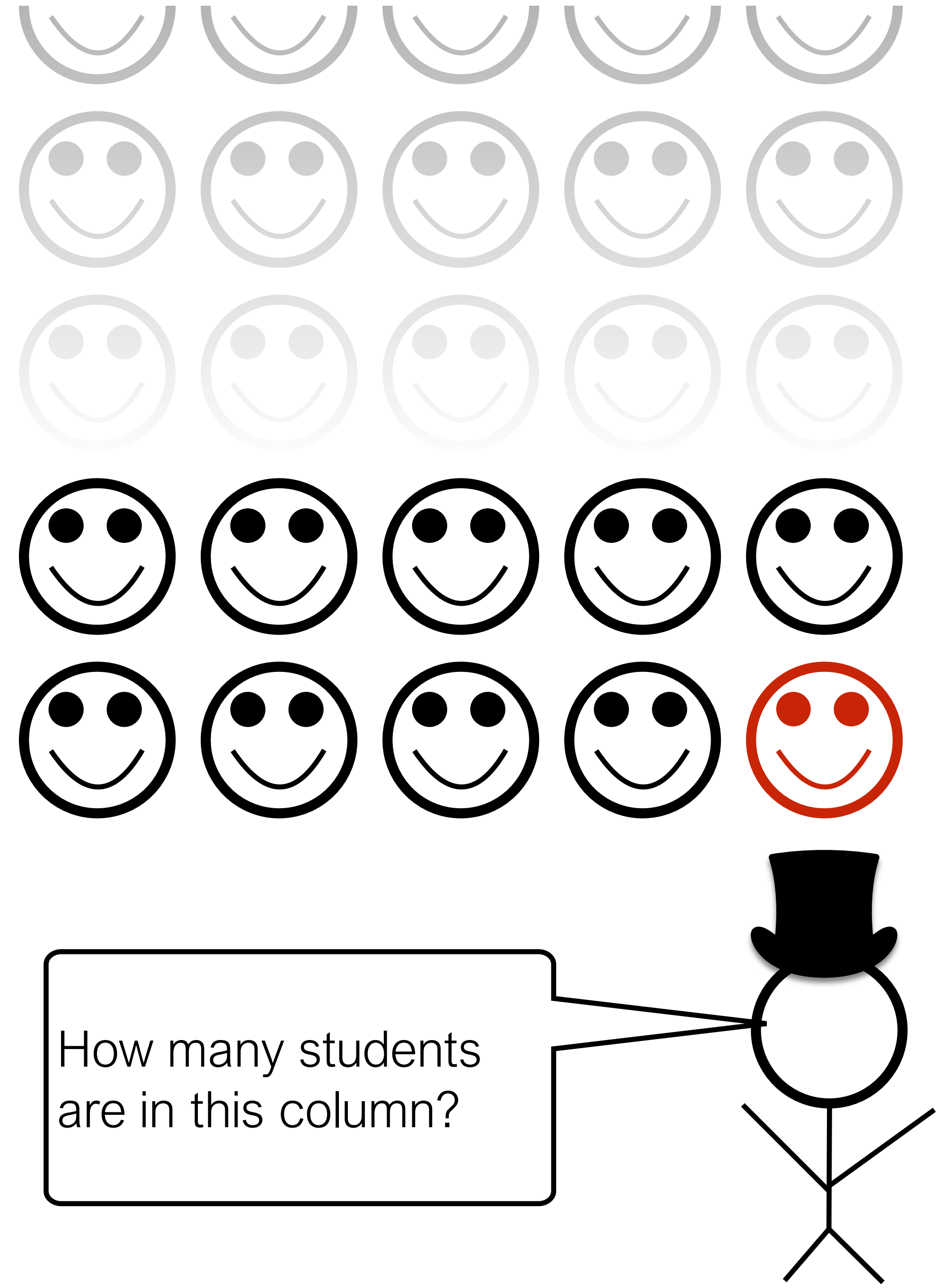
Example from https://courses.cs.washington.edu/courses/cse143/17au/

Recursive Student Counting

Imagine:

A teacher wants to know how many students are in a column.

What should each student ask the person behind them?



Constraints:

- It is dark, you **can't** see the back
- You **can't** get up to count
- You **may** talk to adjacent students
- Mic is broken (students in back can't hear from front)

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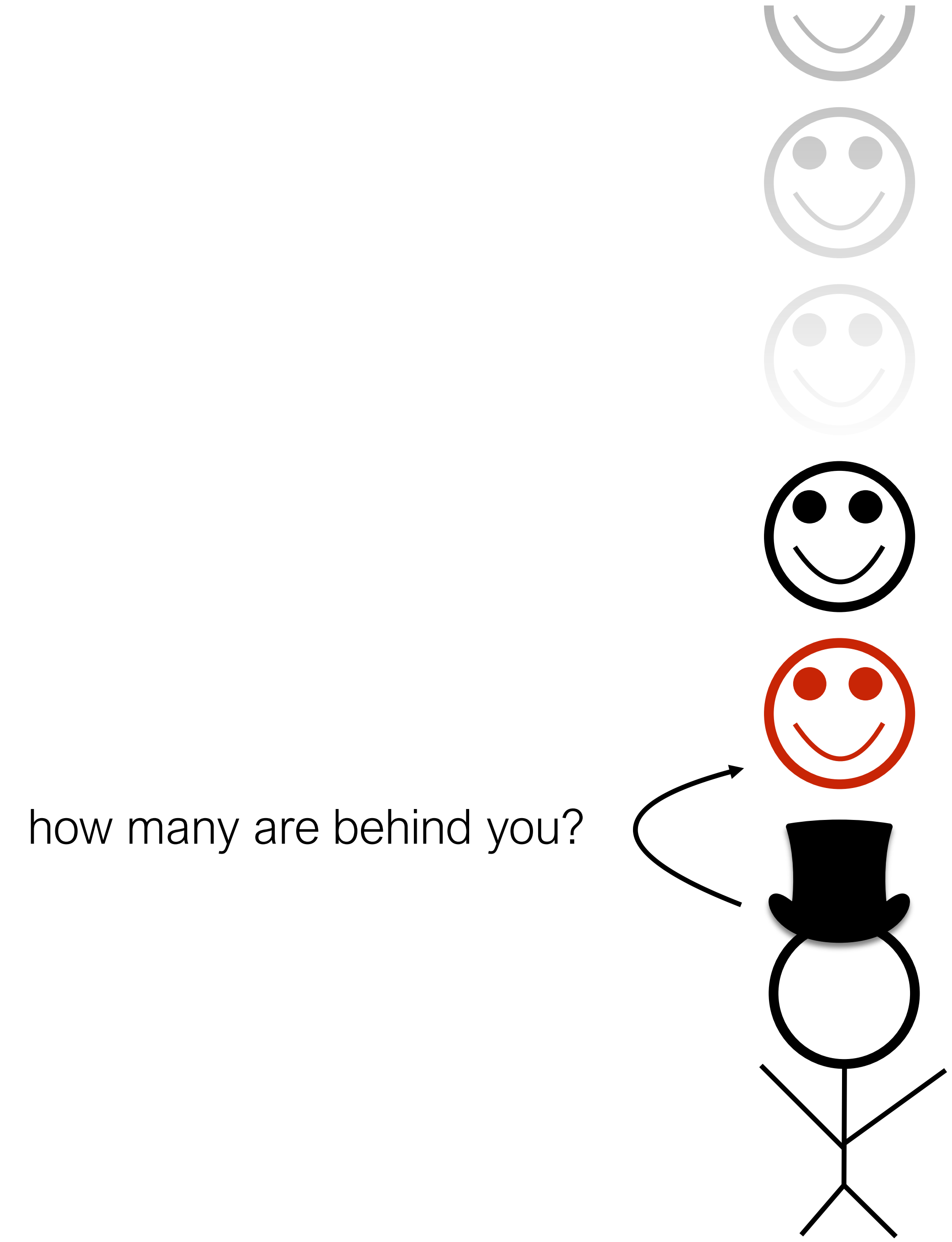
Recursive Student Counting

Strategy: reframe question as *"how many students are behind you?"*



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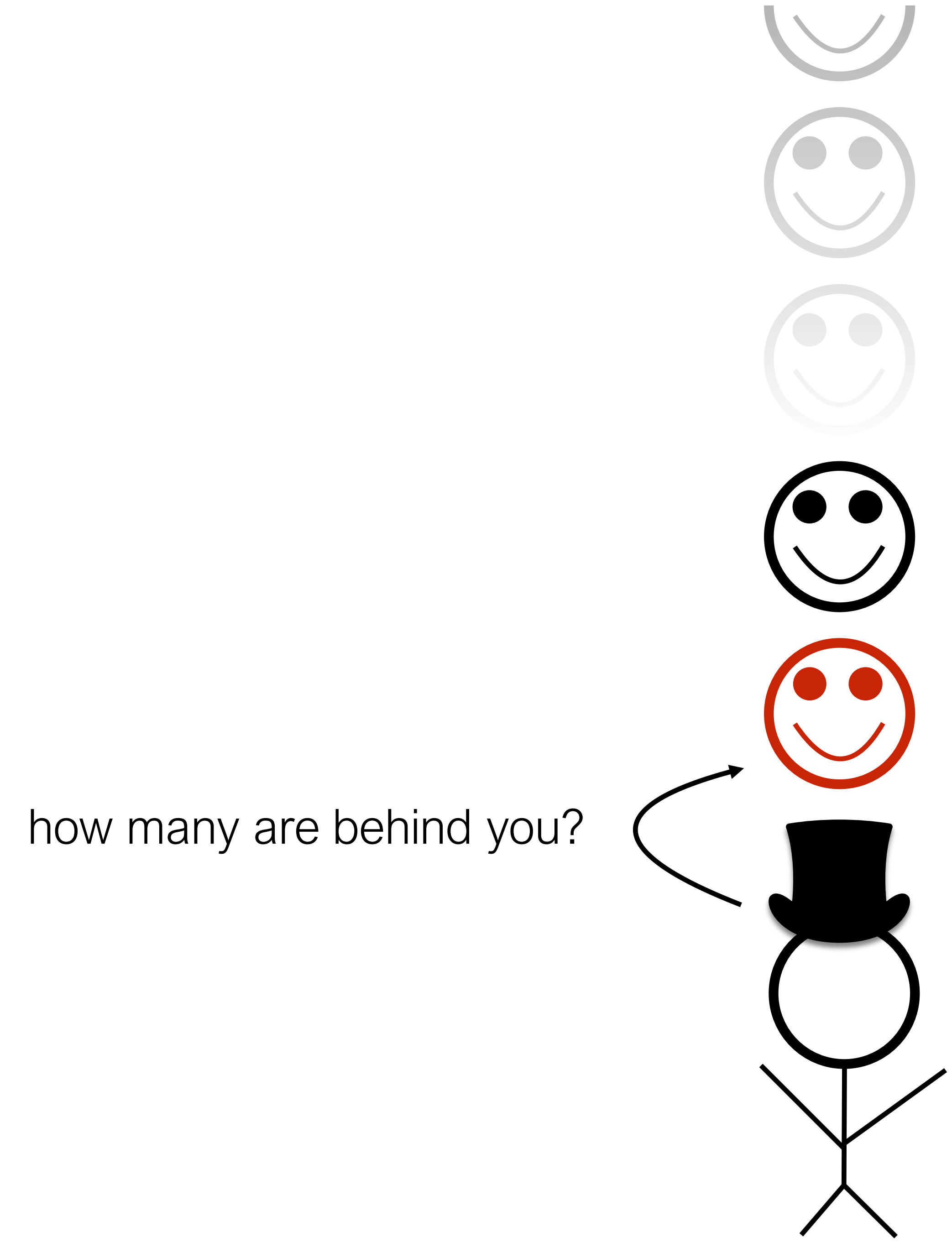
Reframing is the hardest part



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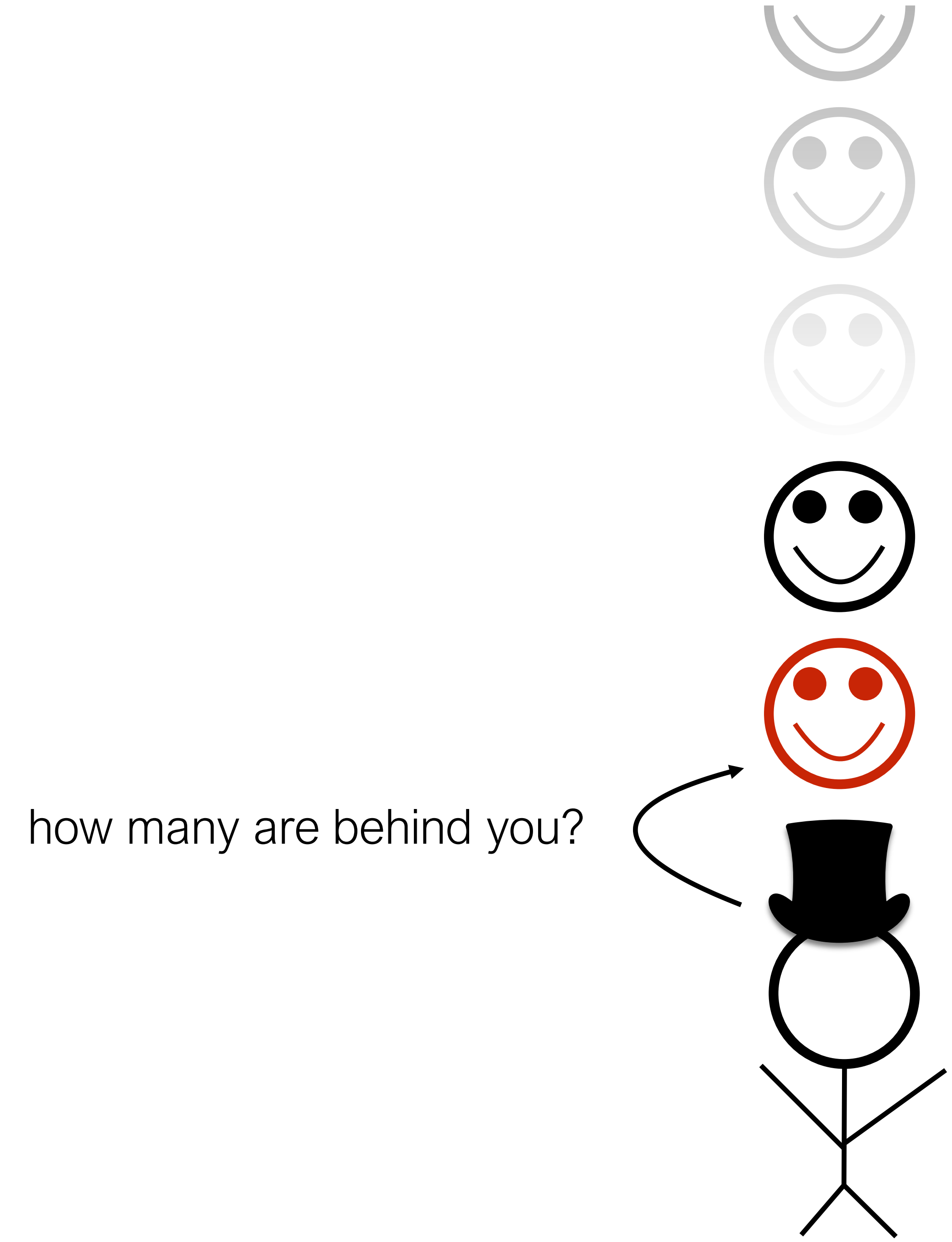
Process:
if nobody is behind you: **say** 0
else: ask them, **say** their answer+1



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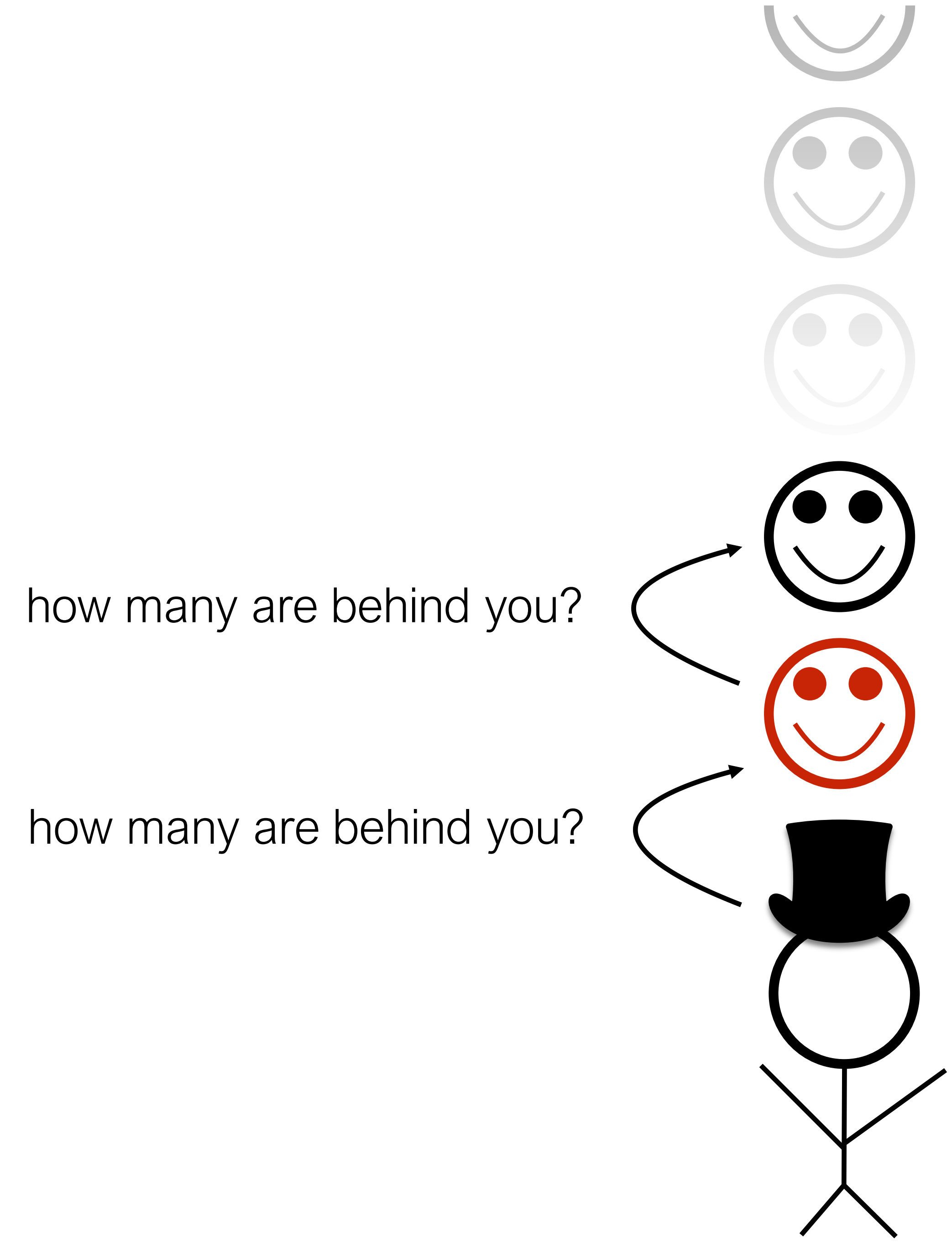

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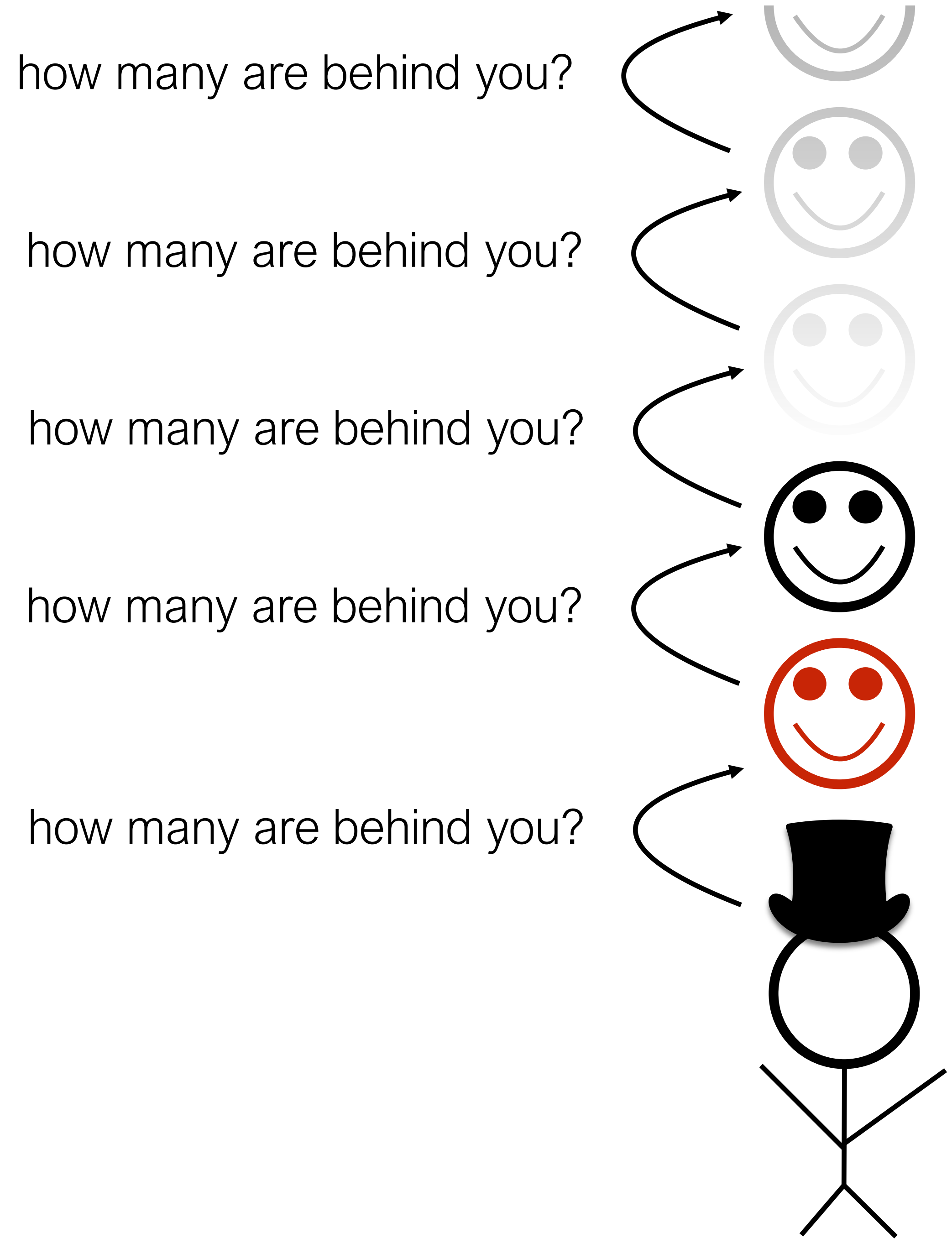

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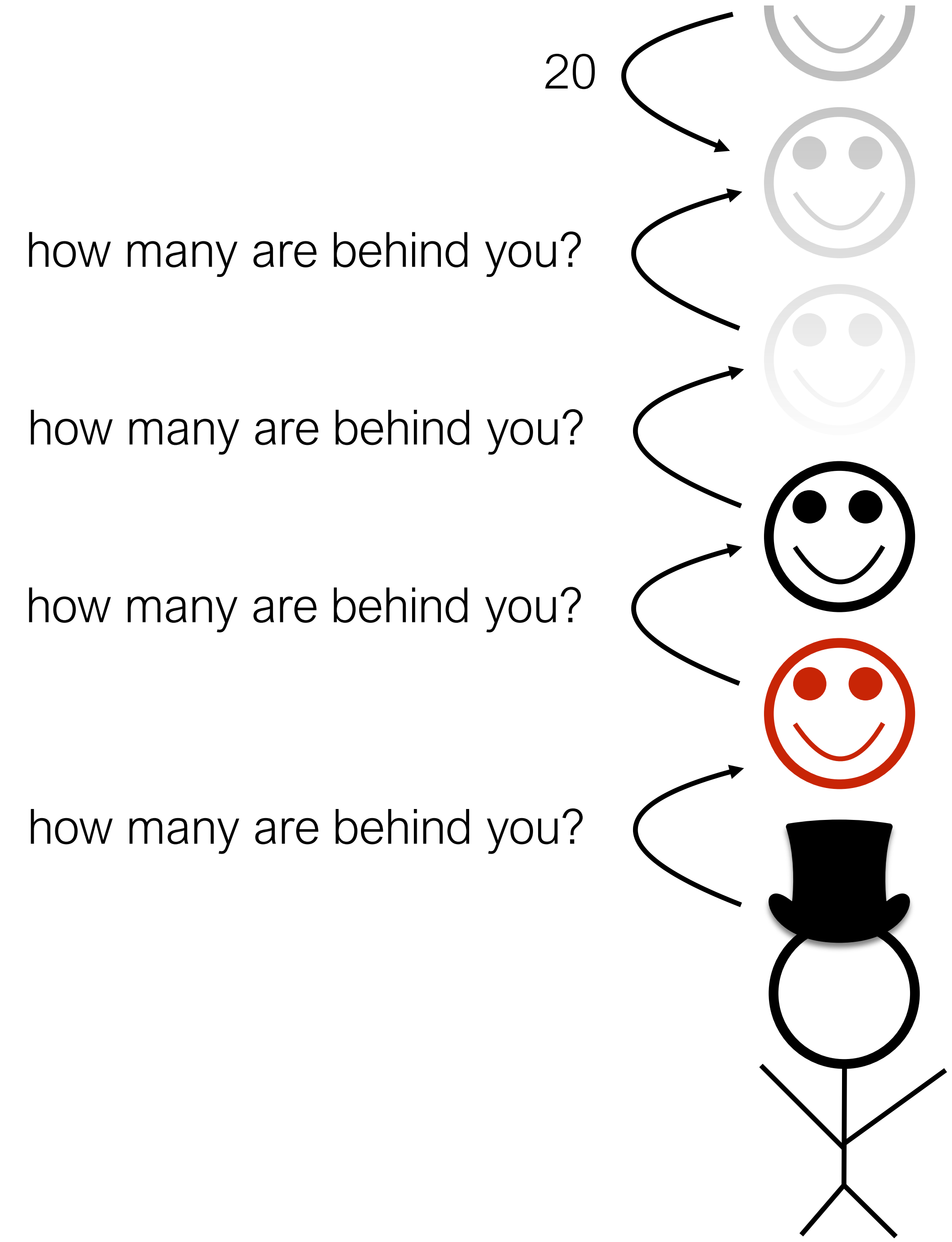

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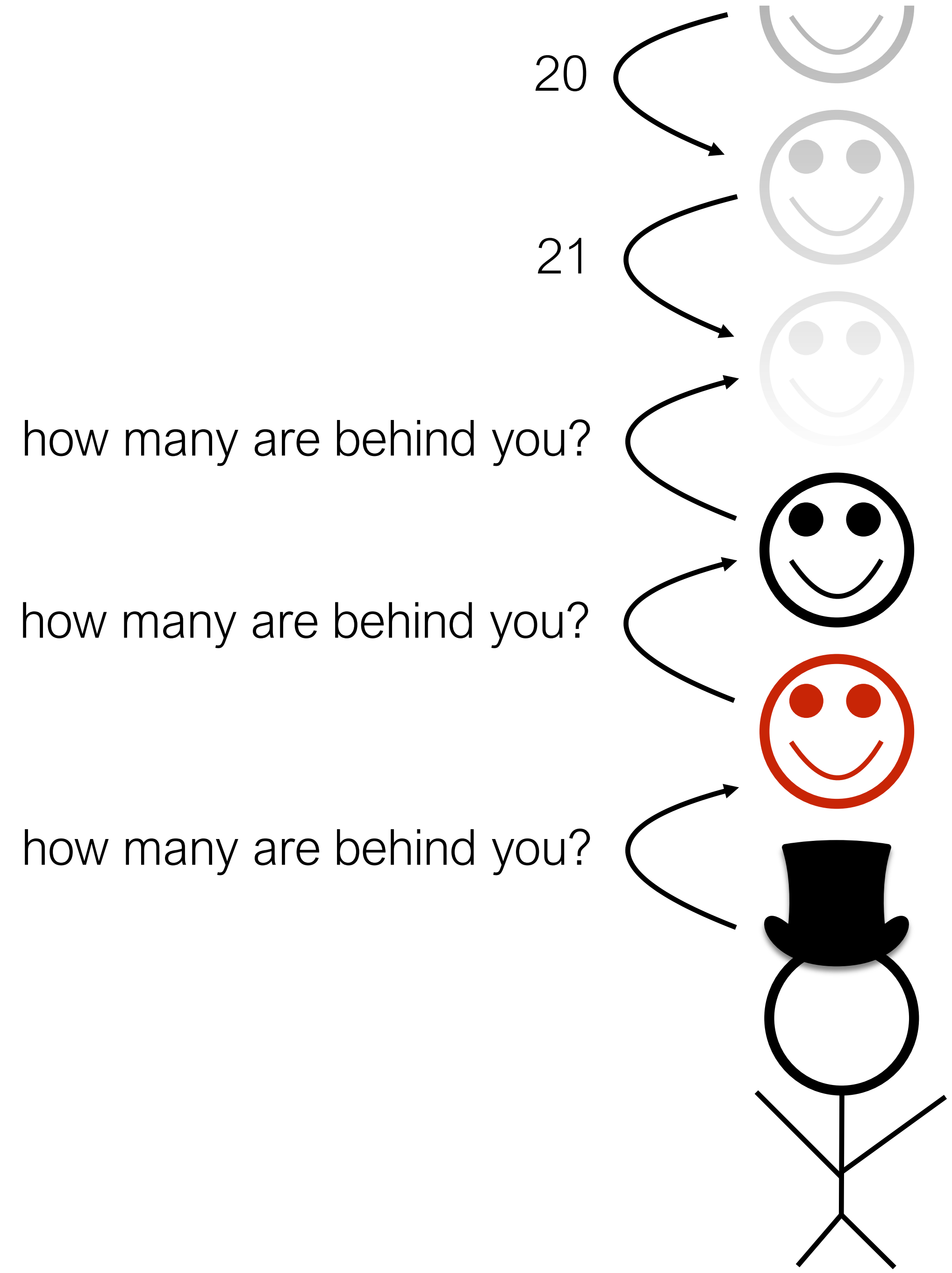

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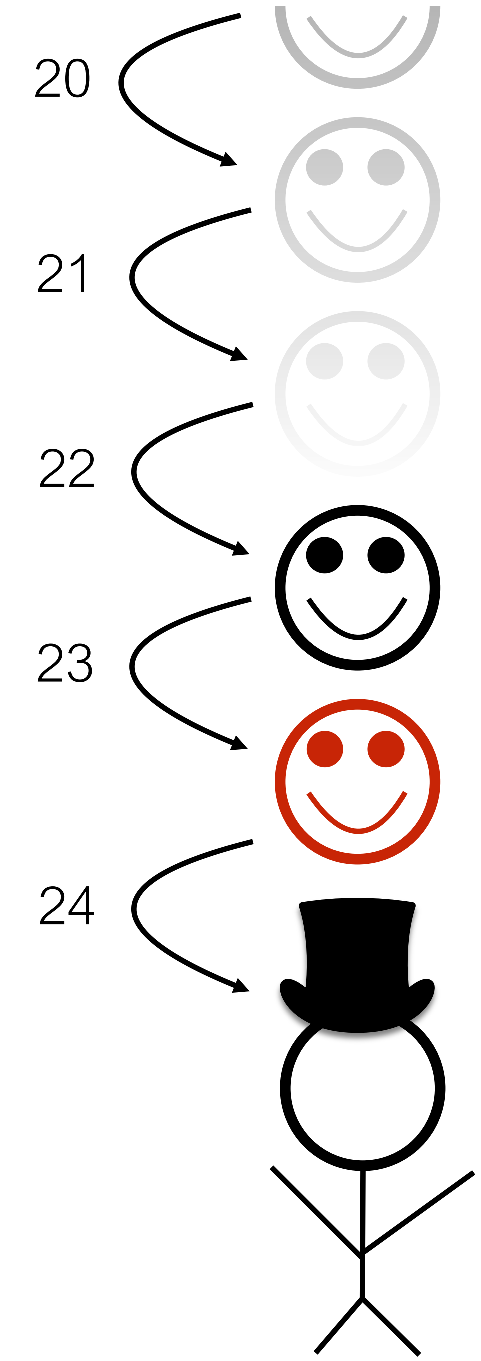

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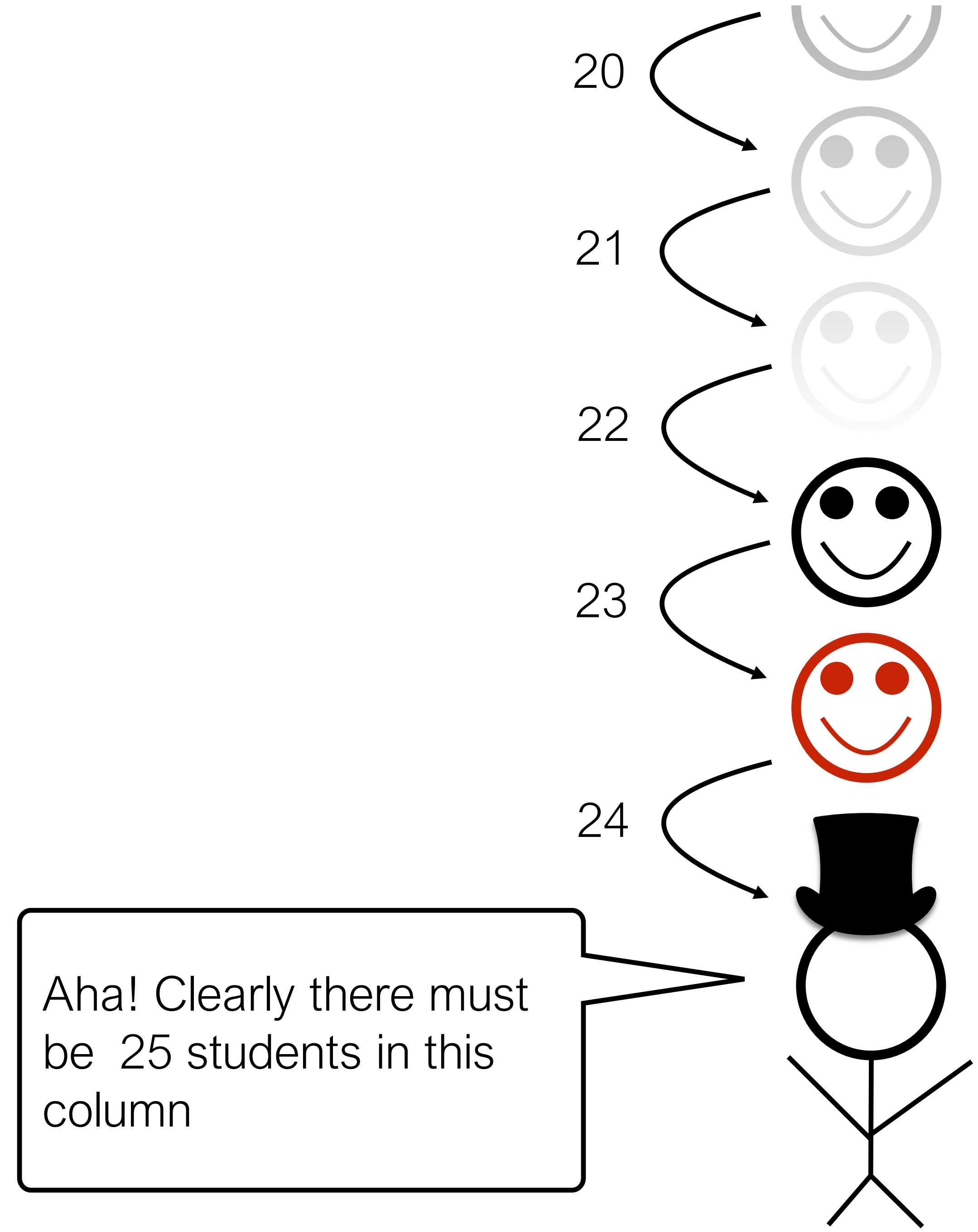

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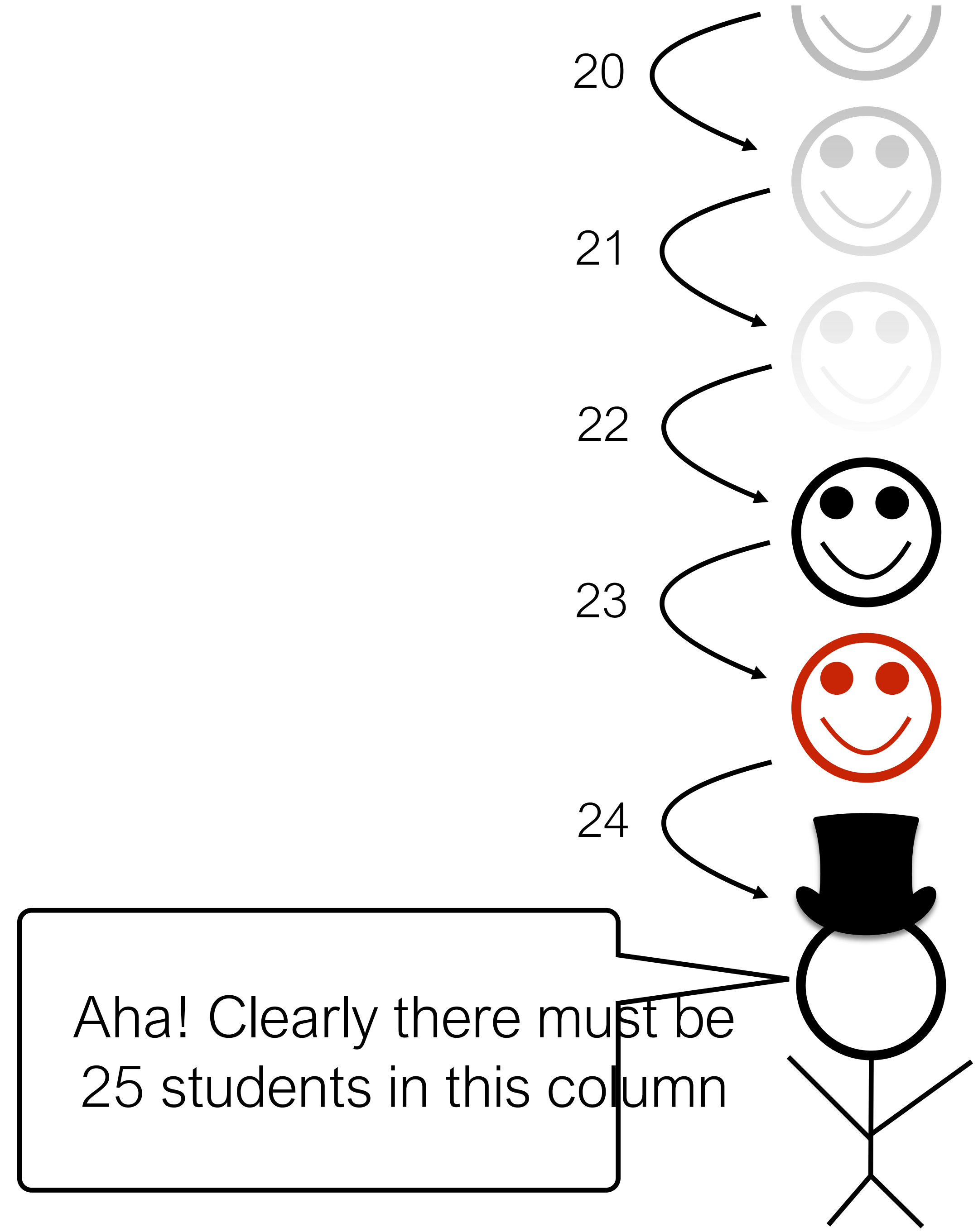
Observations:

- Each student runs the same "code"
- Each student has their own "state"



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Practice: Reframing Factorials

$$N! = 1 \times 2 \times 3 \times \ldots \times (N-2) \times (N-1) \times N$$

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
3! = 1*2*3 = 6
4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

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3. Recursive Definition:

4. Python Code:

```python
def fact(n):
    pass # TODO
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Goal: work from examples to get to recursive code

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```

3. Recursive Definition:

4. Python Code:

```
def fact(n):
    pass # TODO
```

Example: Factorials

1. Examples:

```
1! = 1
2! = 1*2 = 2
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4! = 1*2*3*4 = 24
5! = 1*2*3*4*5 = 120
```

2. Self Reference:

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don't need a pattern at the start

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convert self-referring examples to a recursive definition

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```python
def fact(n):
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```

Let's "run" it!

Tracing Factorial

somebody
called **fact(4)**

```
def fact(n):
    if n == 1:
        return 1
    p = fact(n-1)
    return n * p
```

fact(n=4)

Note, this is **not** a stack frame!
We're tracing code line-by-line.
Boxes represent which invocation.

Tracing Factorial

```
def fact(n):
    if n == 1:
        return 1
    p =fact(n-1)
    return n * p
```

fact(n=4)

if n == 1:

Tracing Factorial

```
def fact(n):
   if n == 1:
      return 1
   p = fact(n-1)
   return n * p
```

fact(n=4)
if n == 1:

Tracing Factorial

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def fact(n):
    if n == 1:
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    p =fact(n-1)
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```

fact(n=4)

if n == 1:

fact(n=3)

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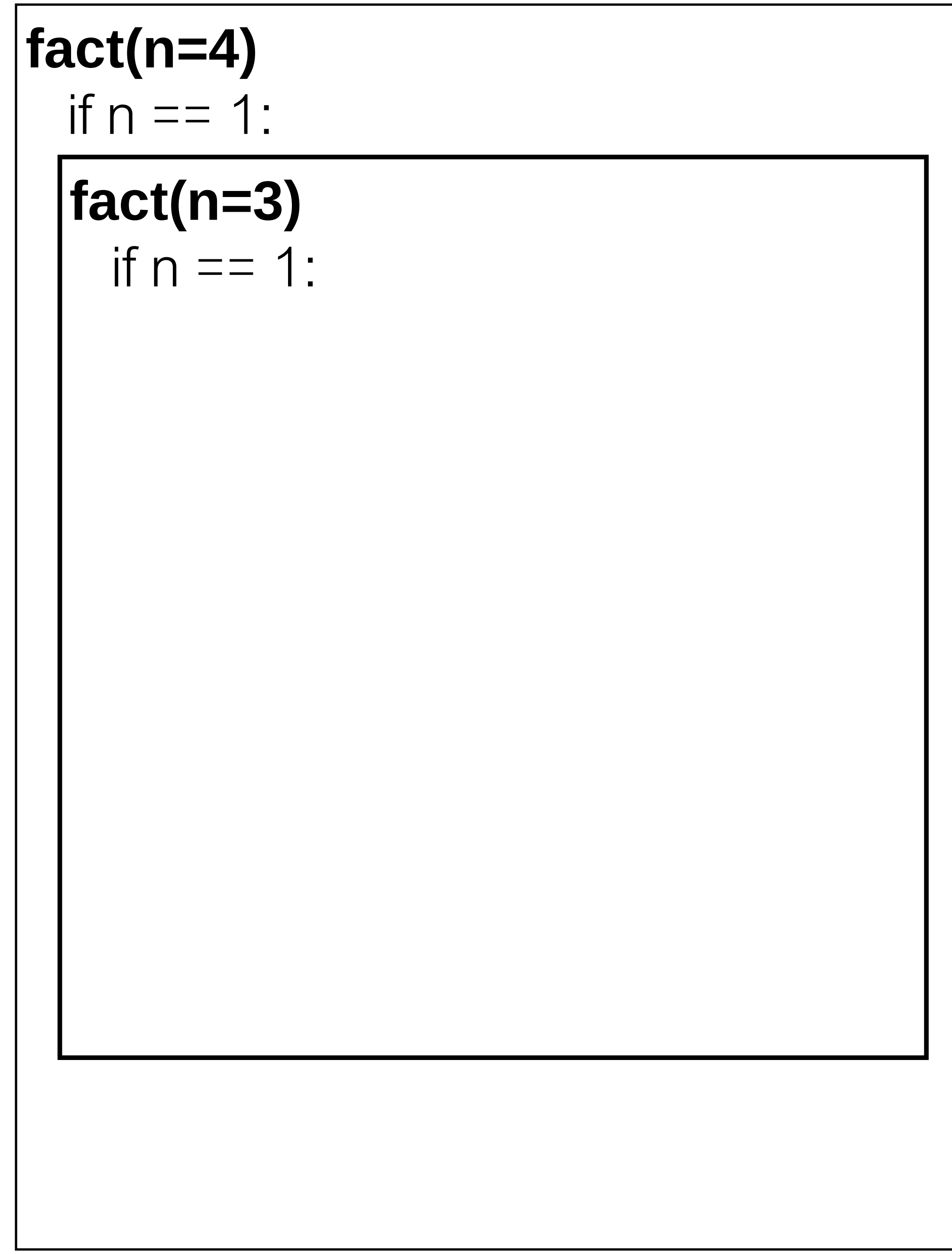

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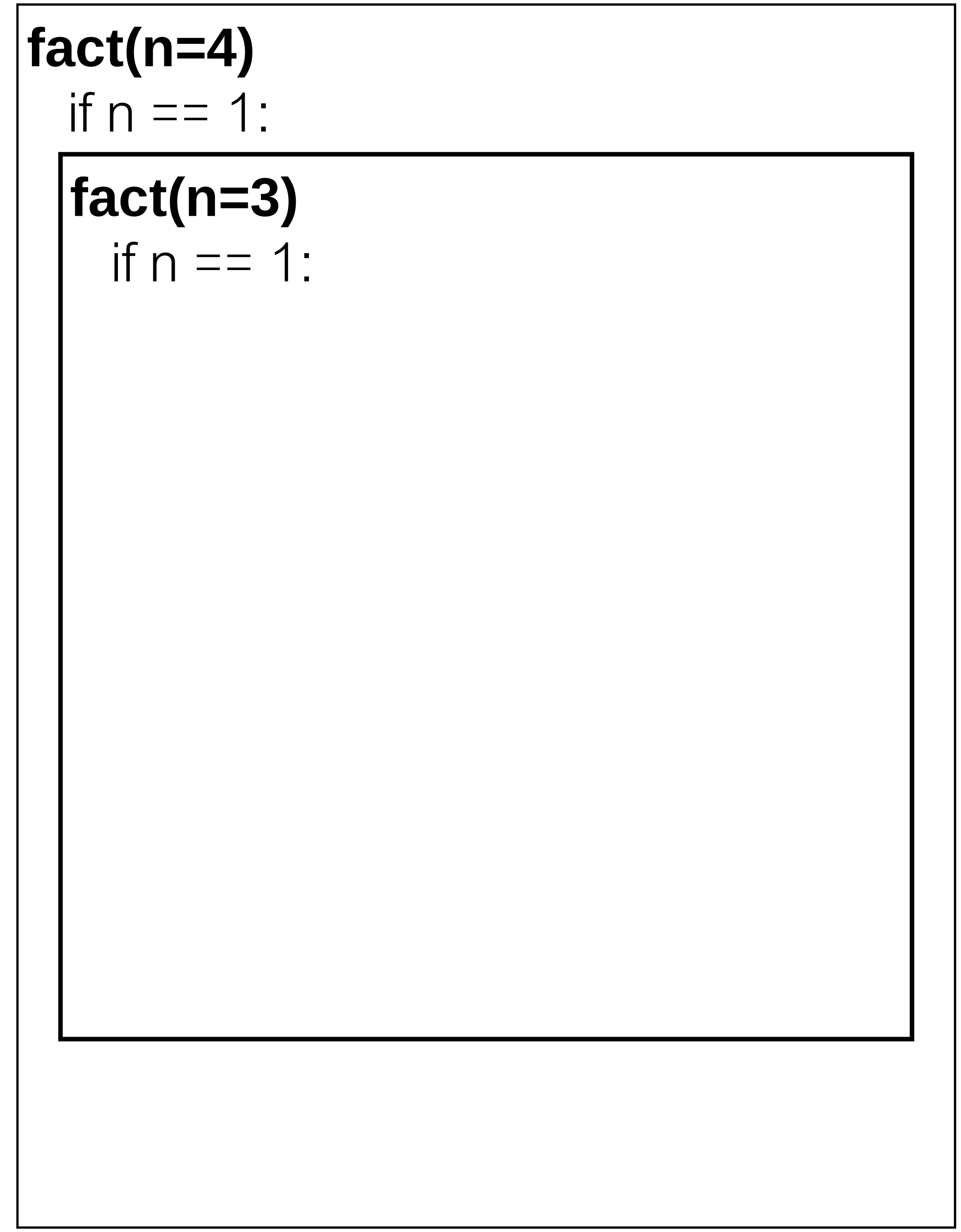

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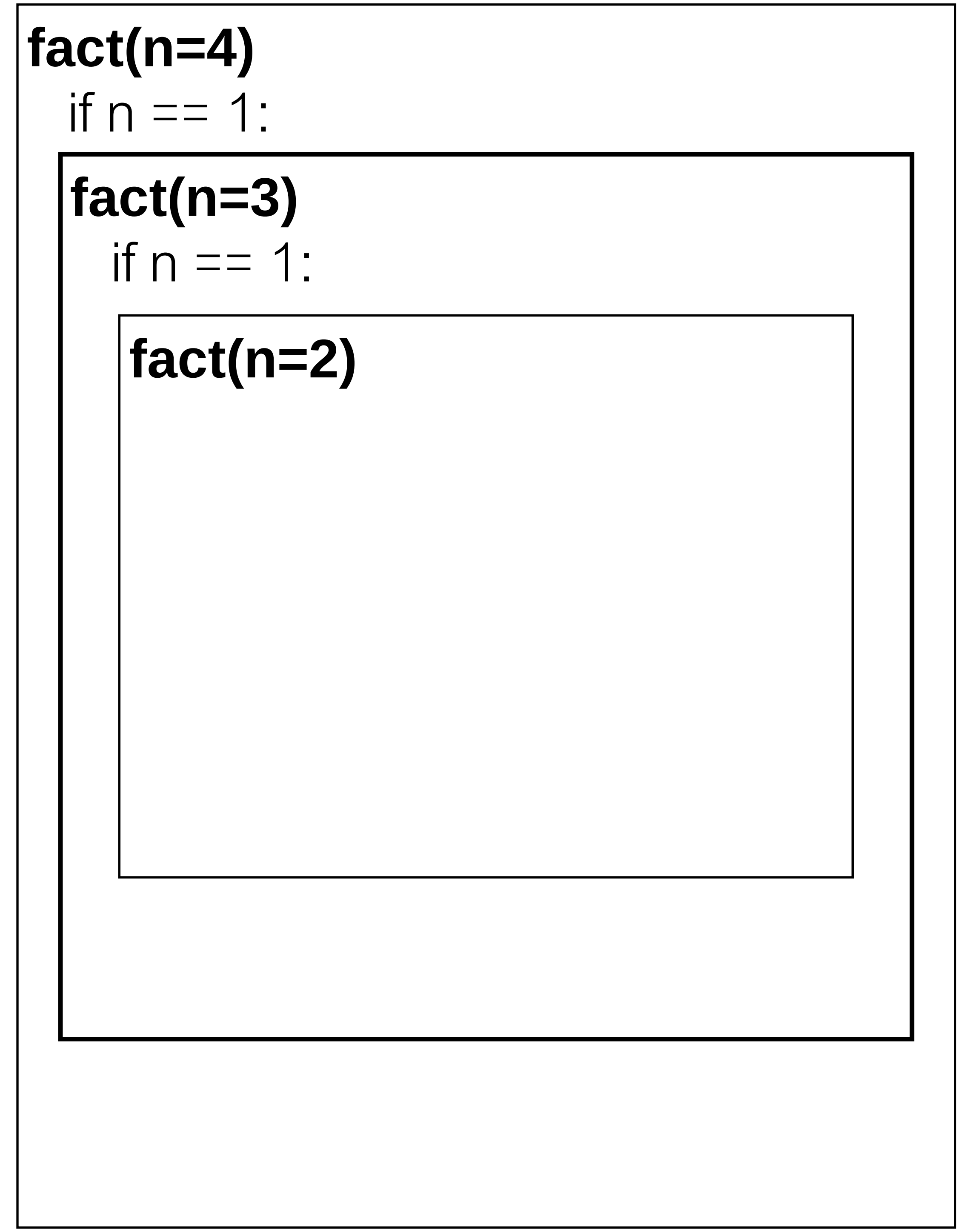

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fact(n=4)

if n == 1:

fact(n=3)

if n == 1:

fact(n=2)

if n == 1:

Tracing Factorial

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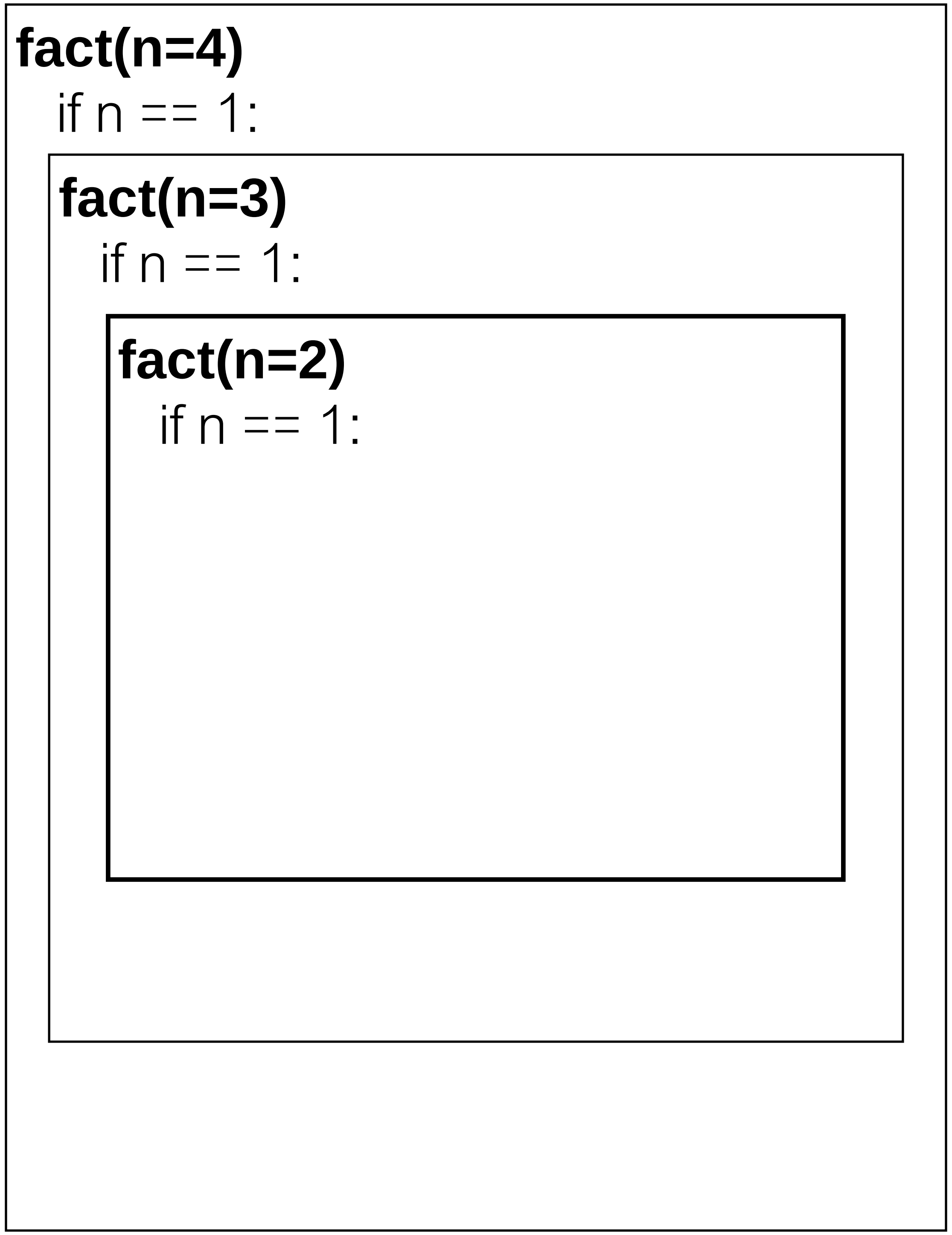

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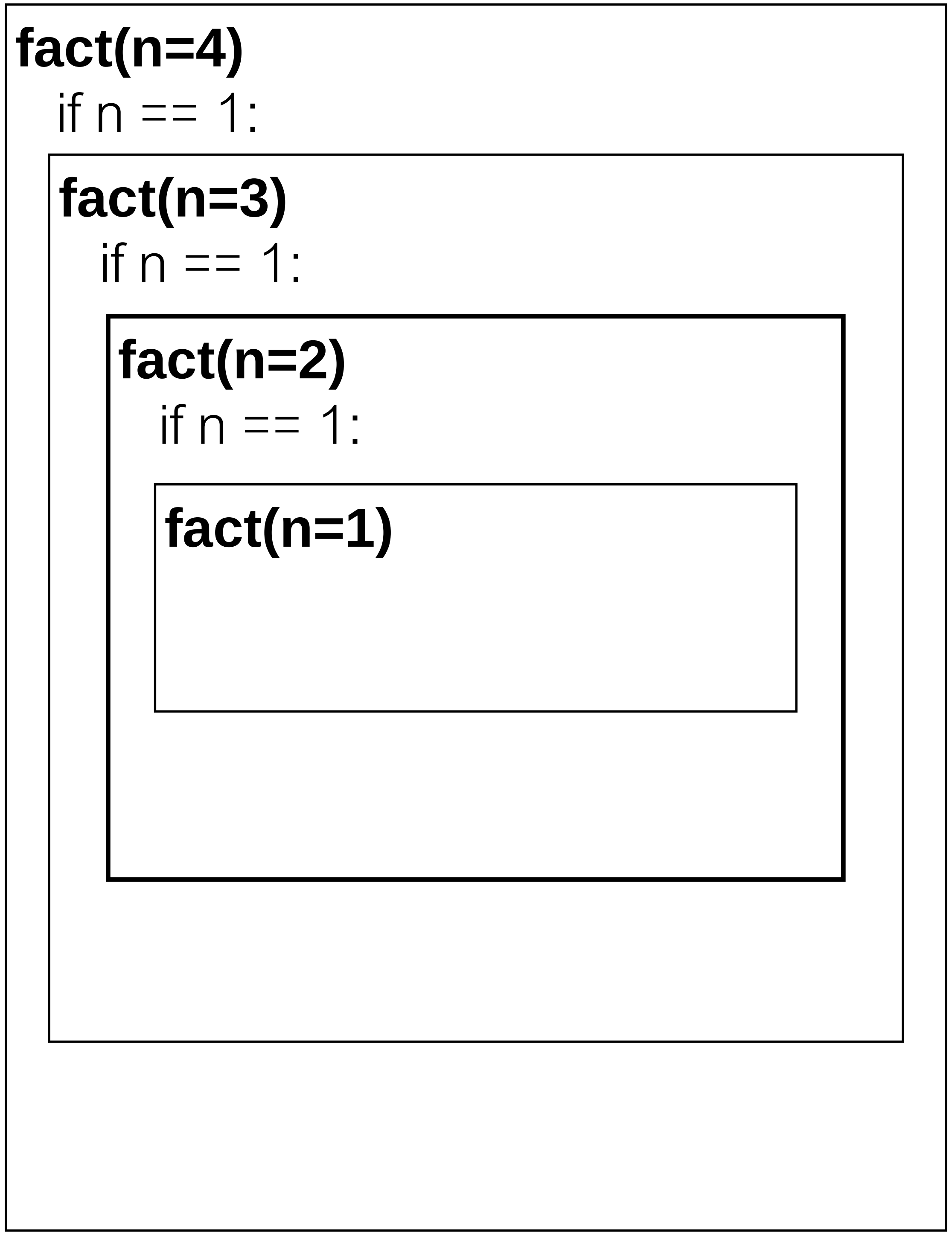

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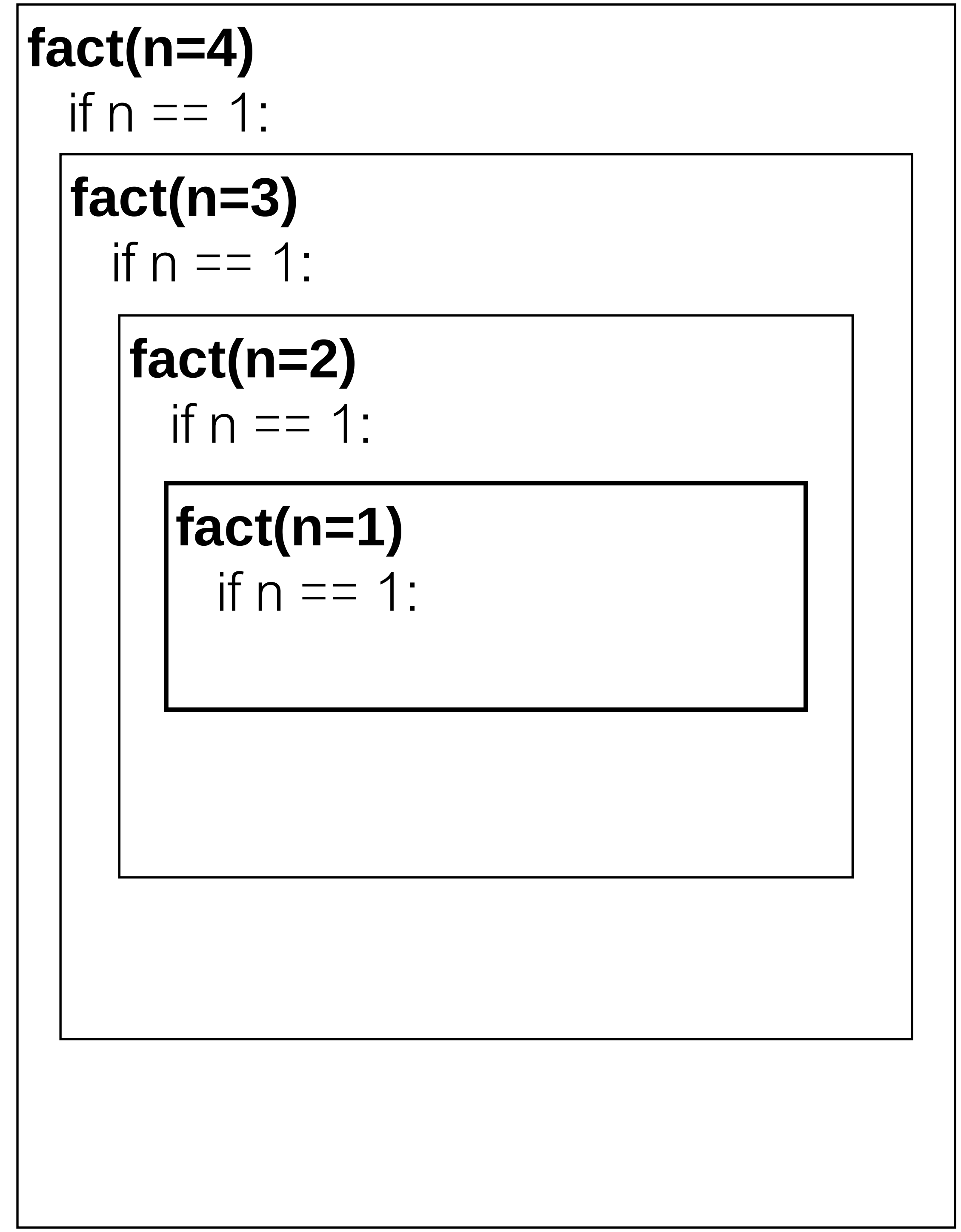

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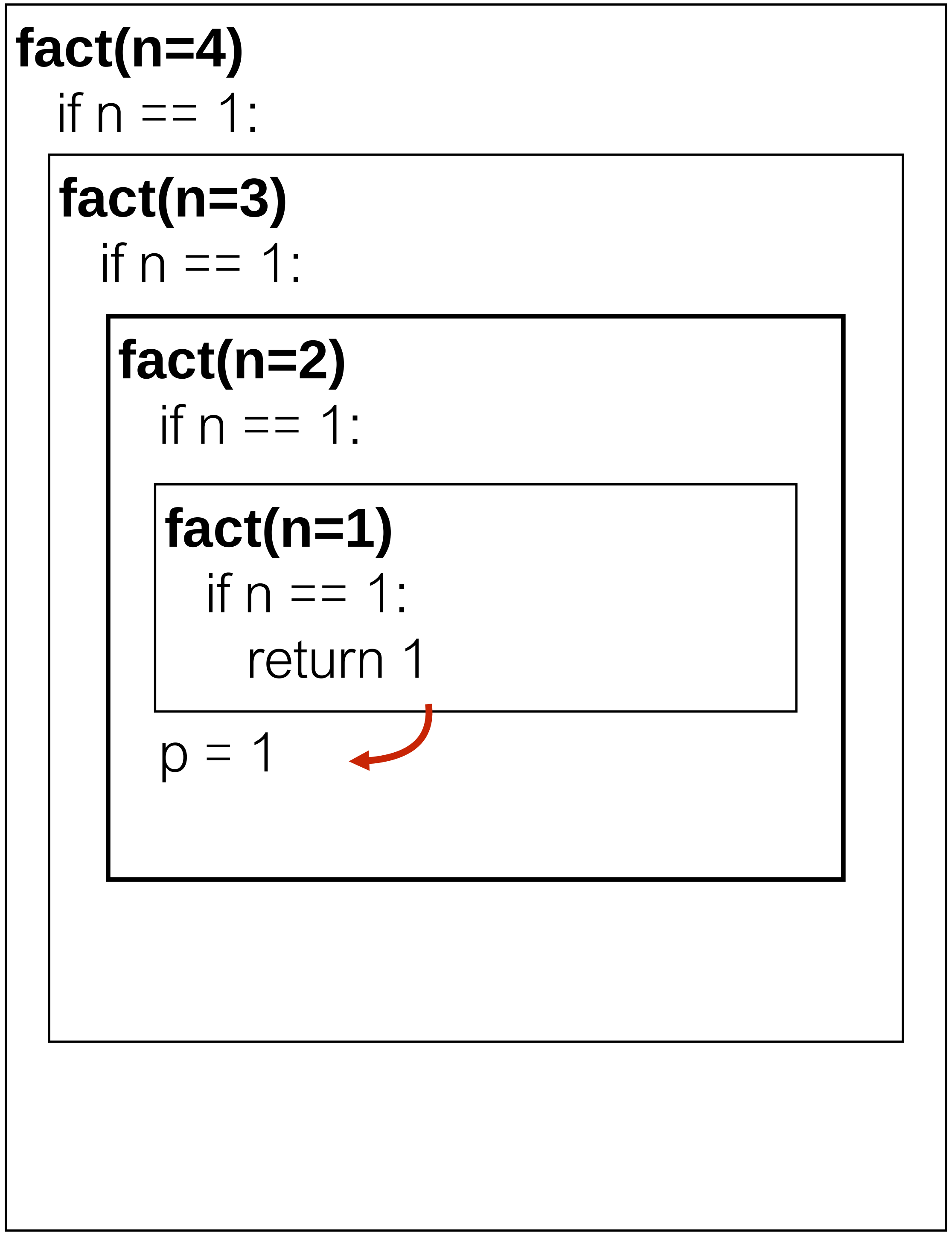

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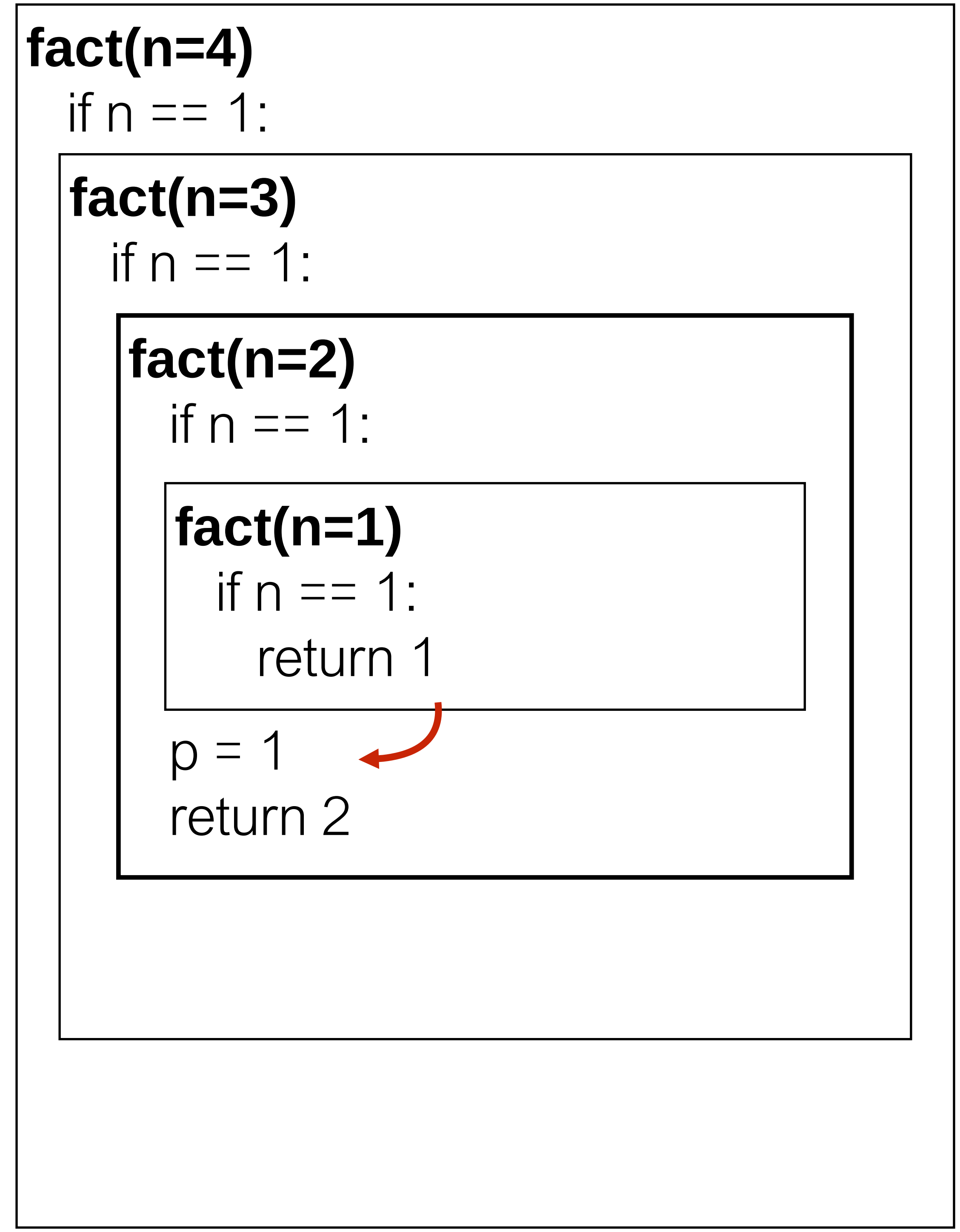

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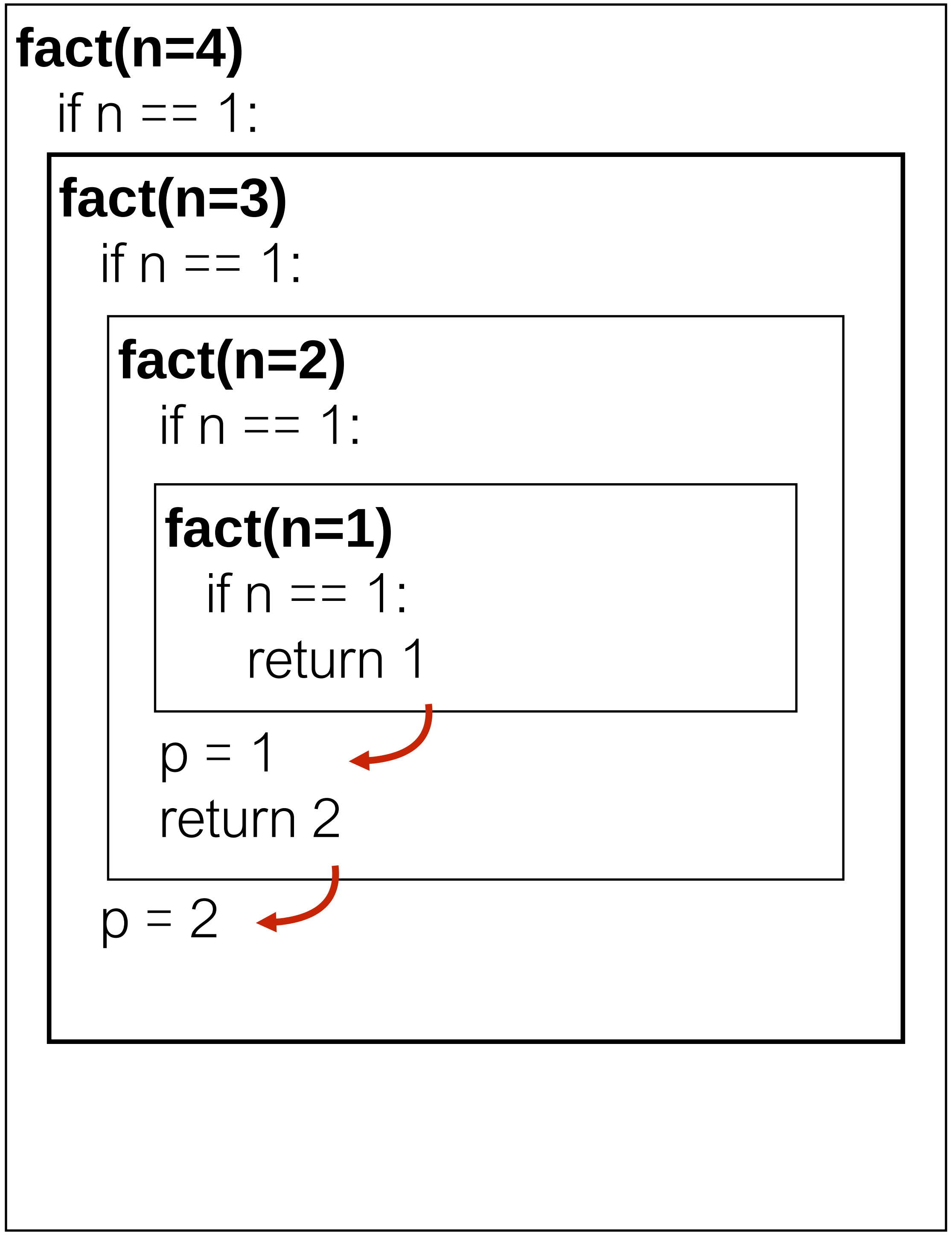

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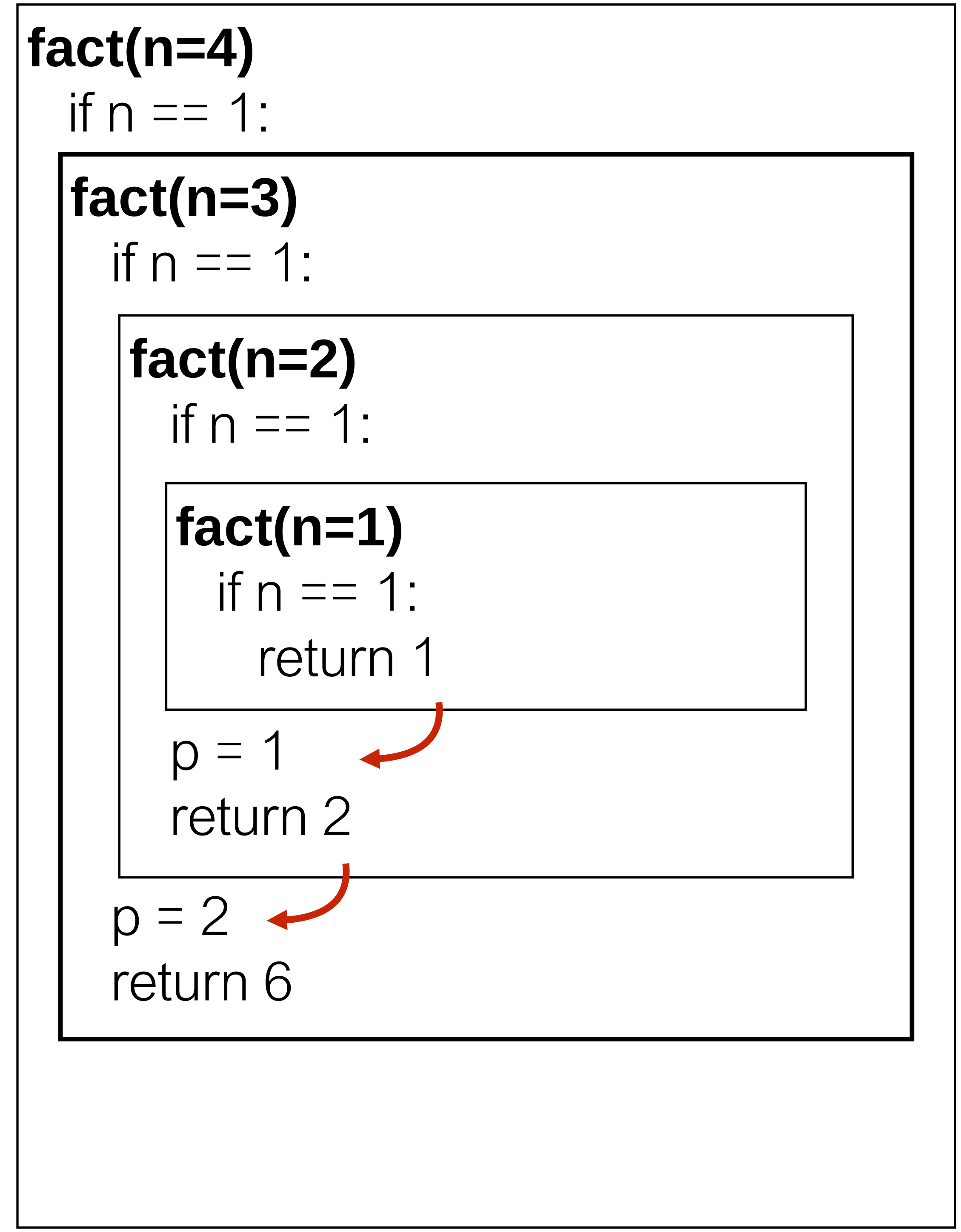

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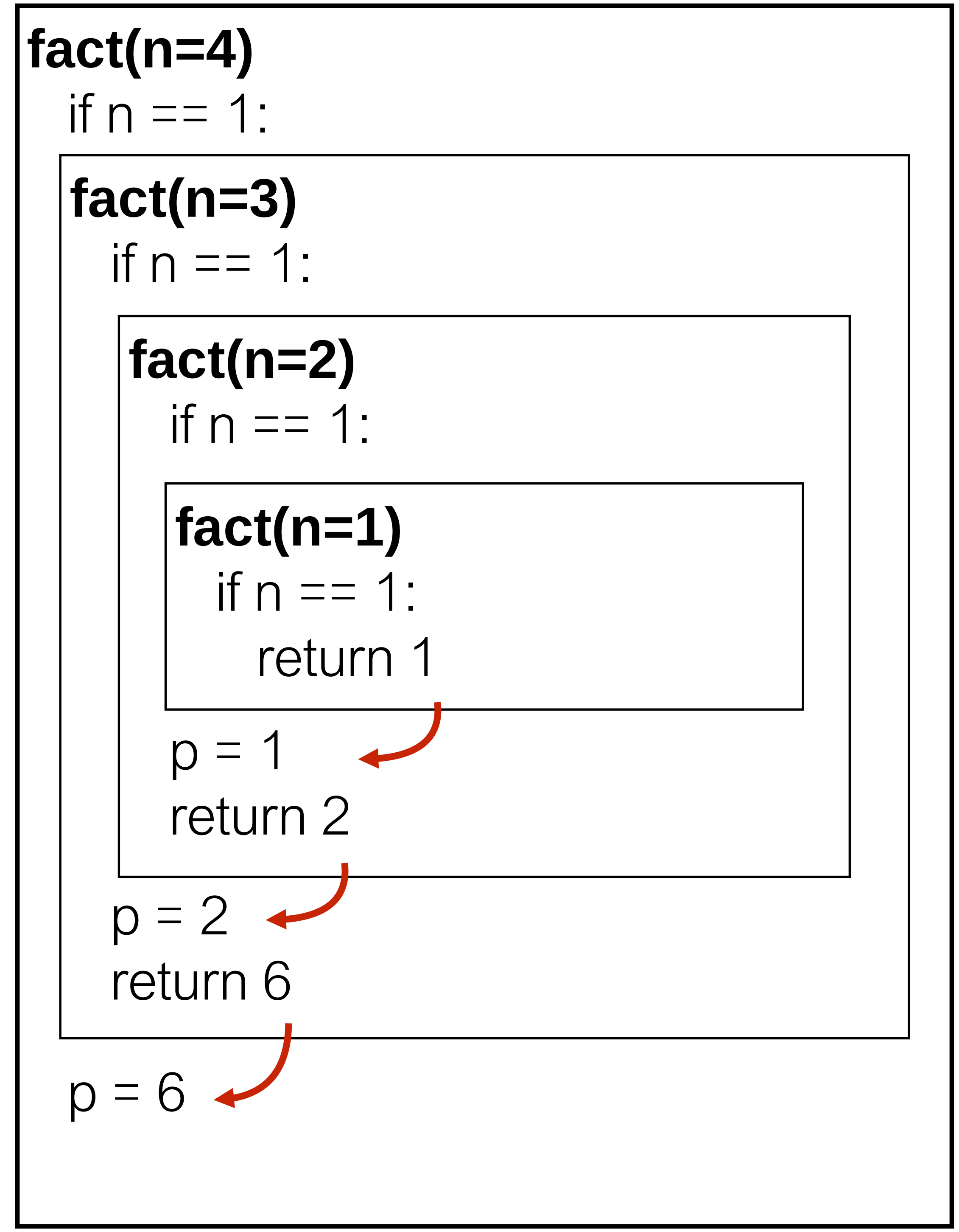

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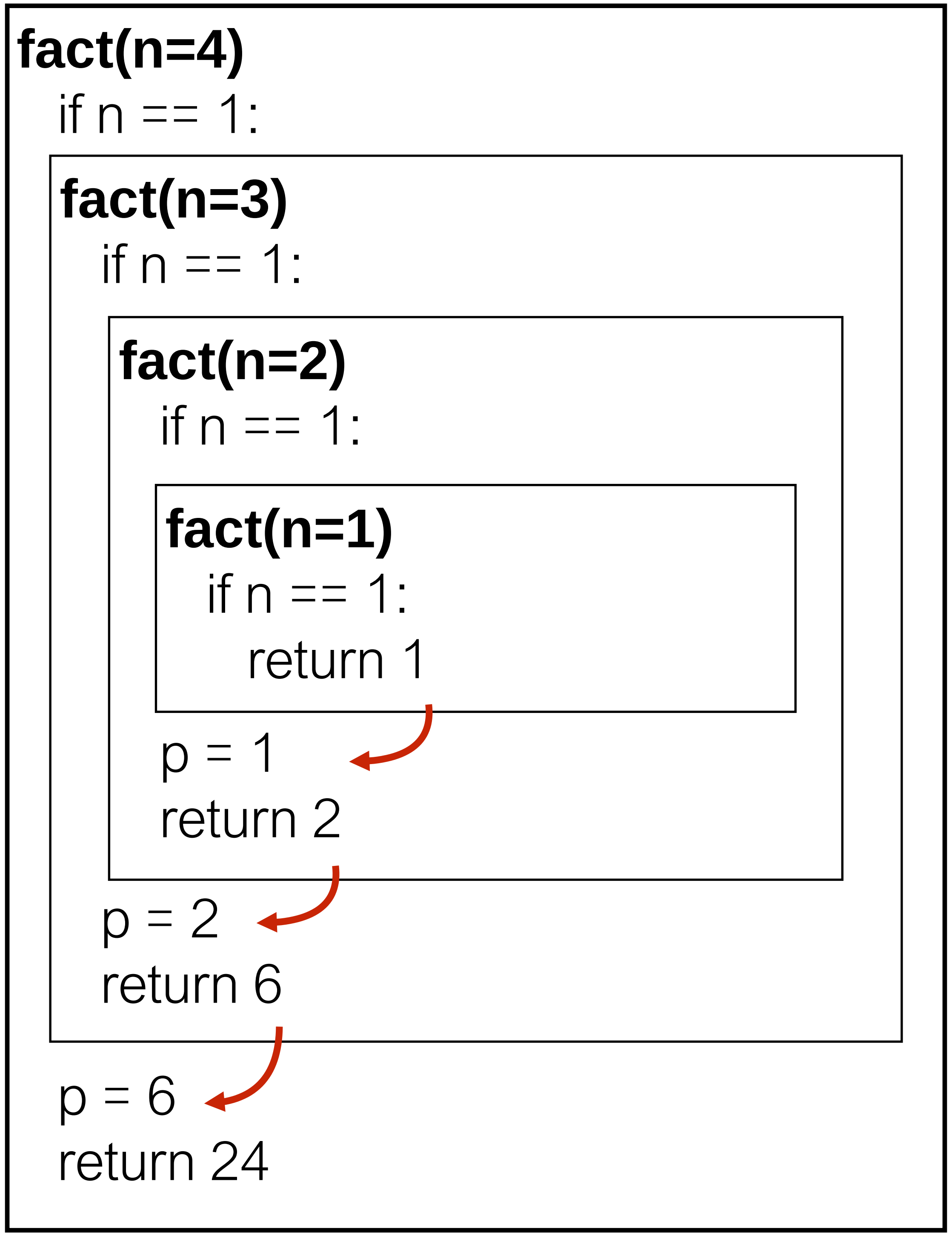

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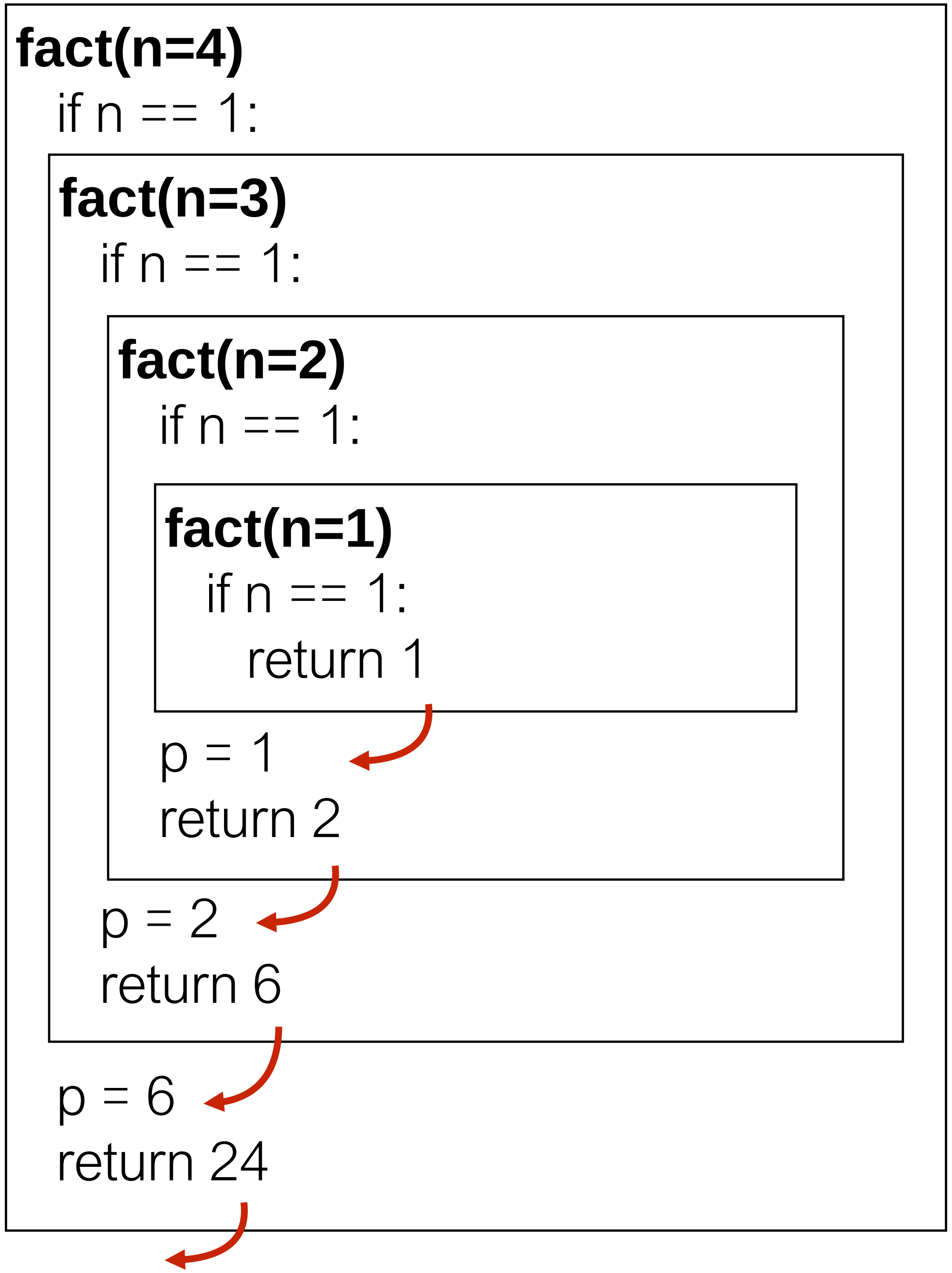

Tracing Factorial

```
def fact(n):
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    return n * p
```

How does Python keep
all the variables separate?

fact(n=4)
if n == 1:

fact(n=3)
if n == 1:

fact(n=2)
if n == 1:

fact(n=1)
if n == 1:
return 1

p = 1
return 2

p = 2
return 6

p = 6
return 24

Tracing Factorial

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def fact(n):
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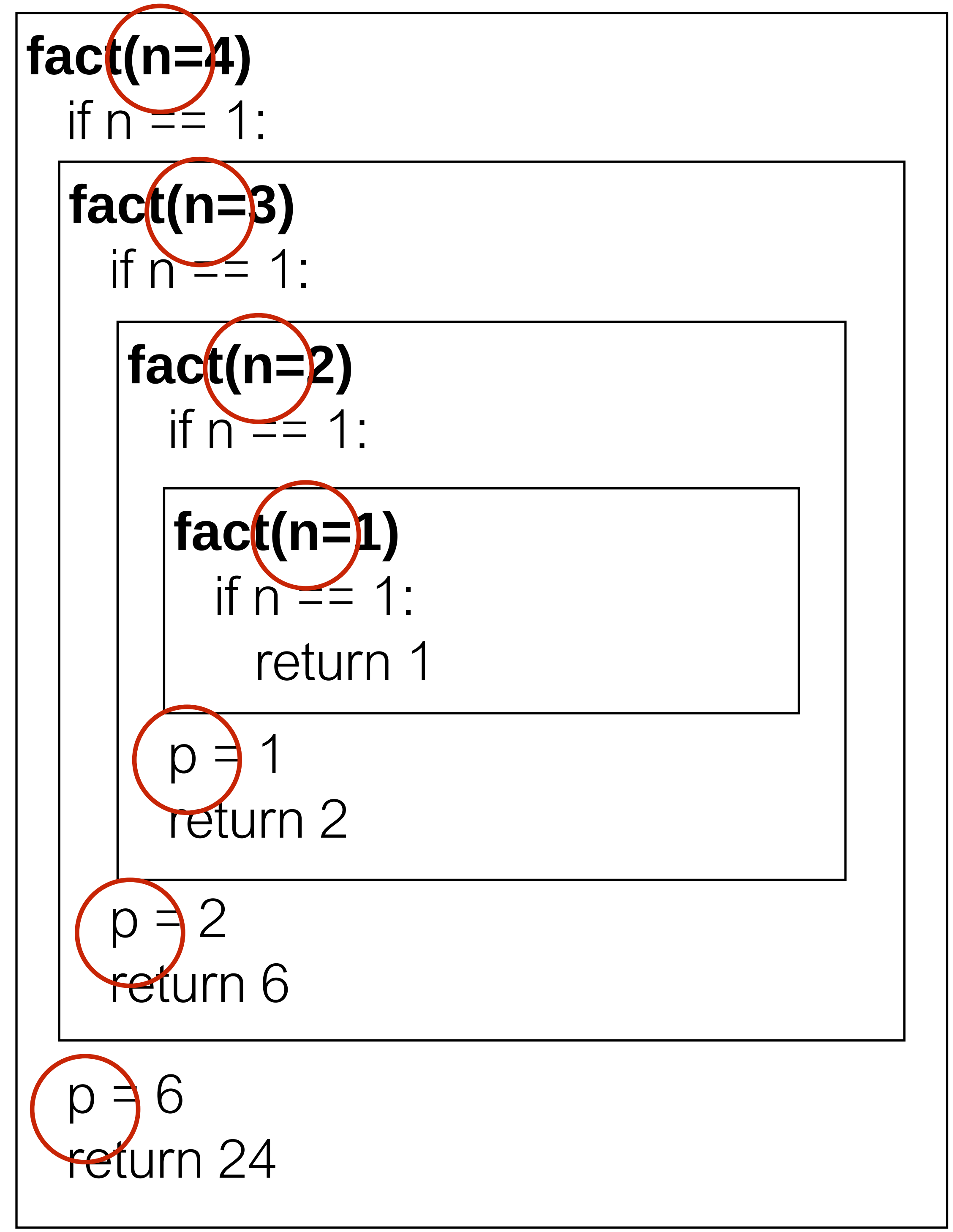

How does Python keep all the variables separate?

frames to the rescue!

Deep Dive: Invocation State

In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Deep Dive: Invocation State

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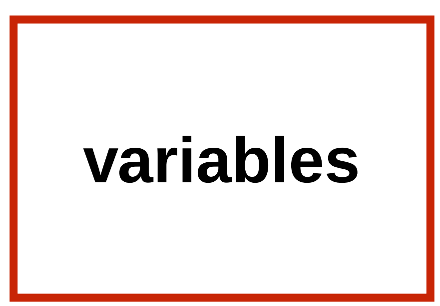
Variables for an invocation exist in a ***frame***

frame:



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Variables for an invocation exist in a ***frame***

- the frames are stored in the **stack**

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In recursion, each function invocation has its **own state**, but multiple invocations **share code**.

Variables for an invocation exist in a ***frame***

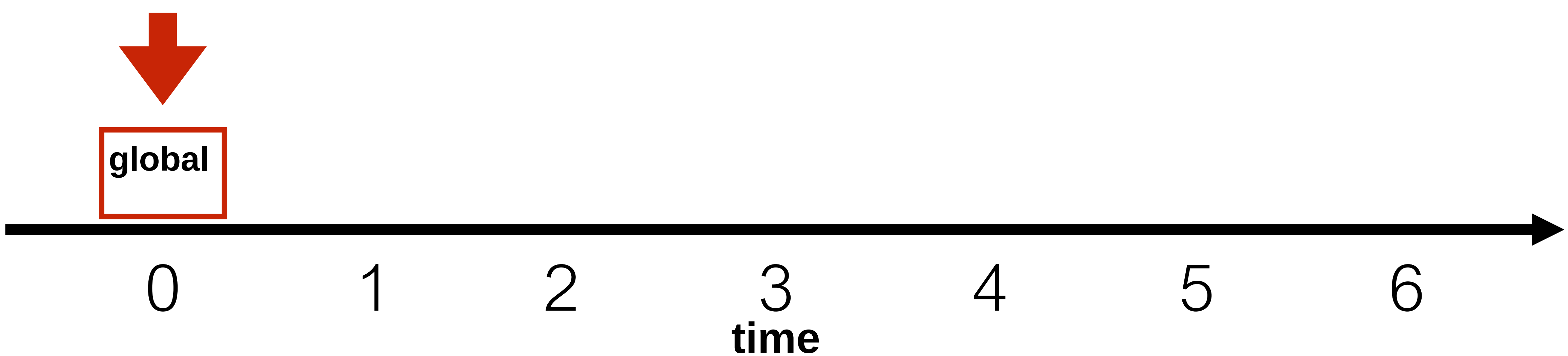
- the frames are stored in the **stack**
- one invocation is active at a time: its frame is on the top of stack
- if a function calls itself, there will be multiple frames at the same time for the multiple invocations of the same function

Deep Dive: Runtime Stack

```
def fact(n):
    if n == 1:
        return 1
    p =fact(n-1)
    return n * p
```

call `fact(3)`

Current Runtime Stack



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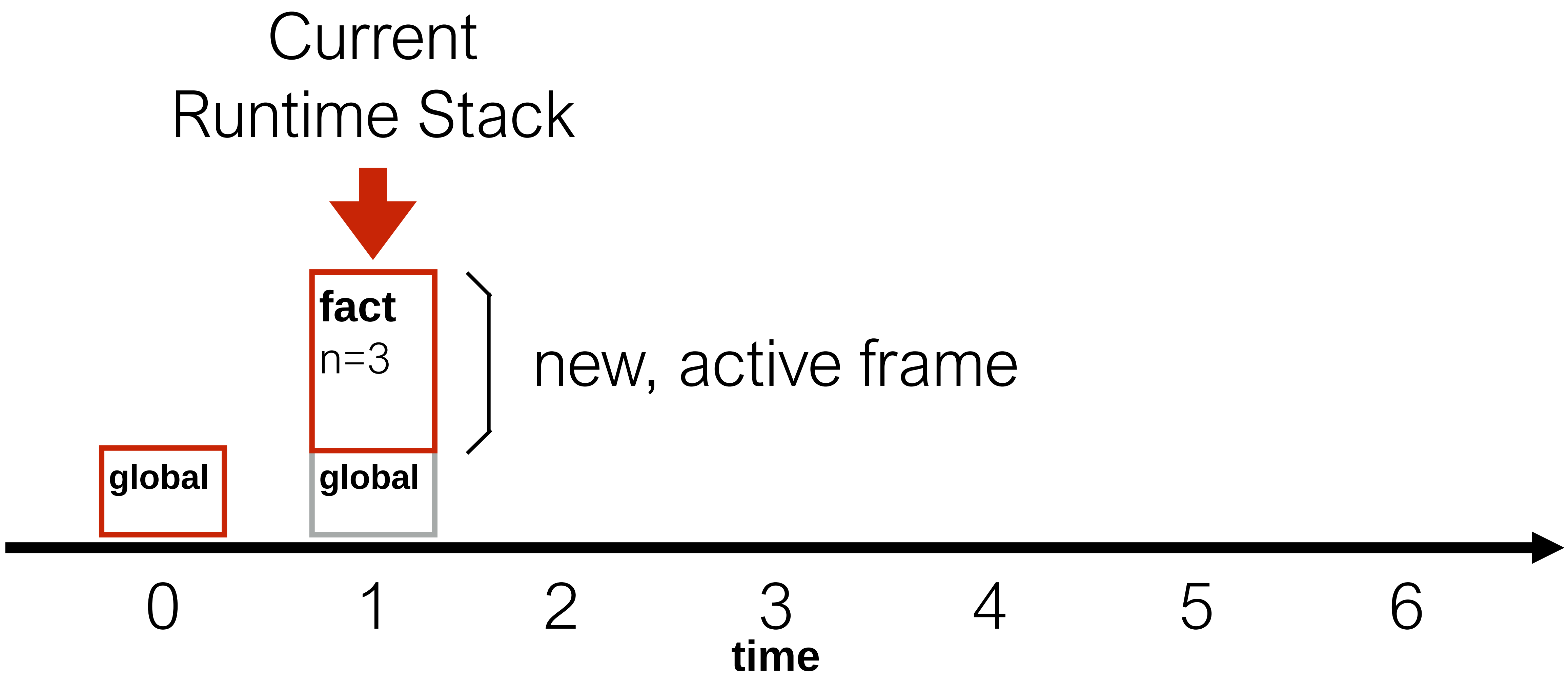

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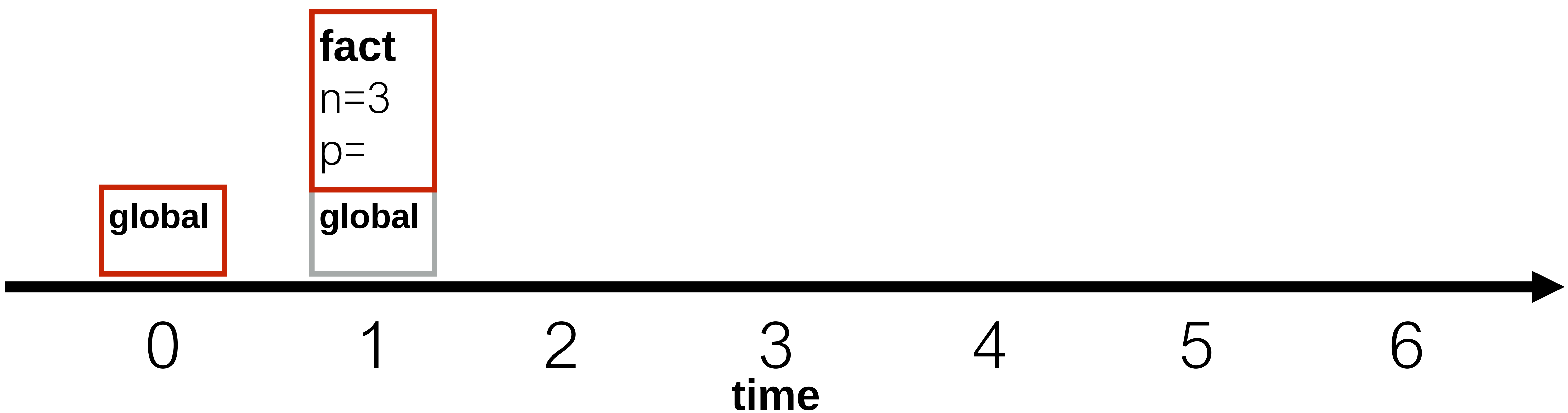

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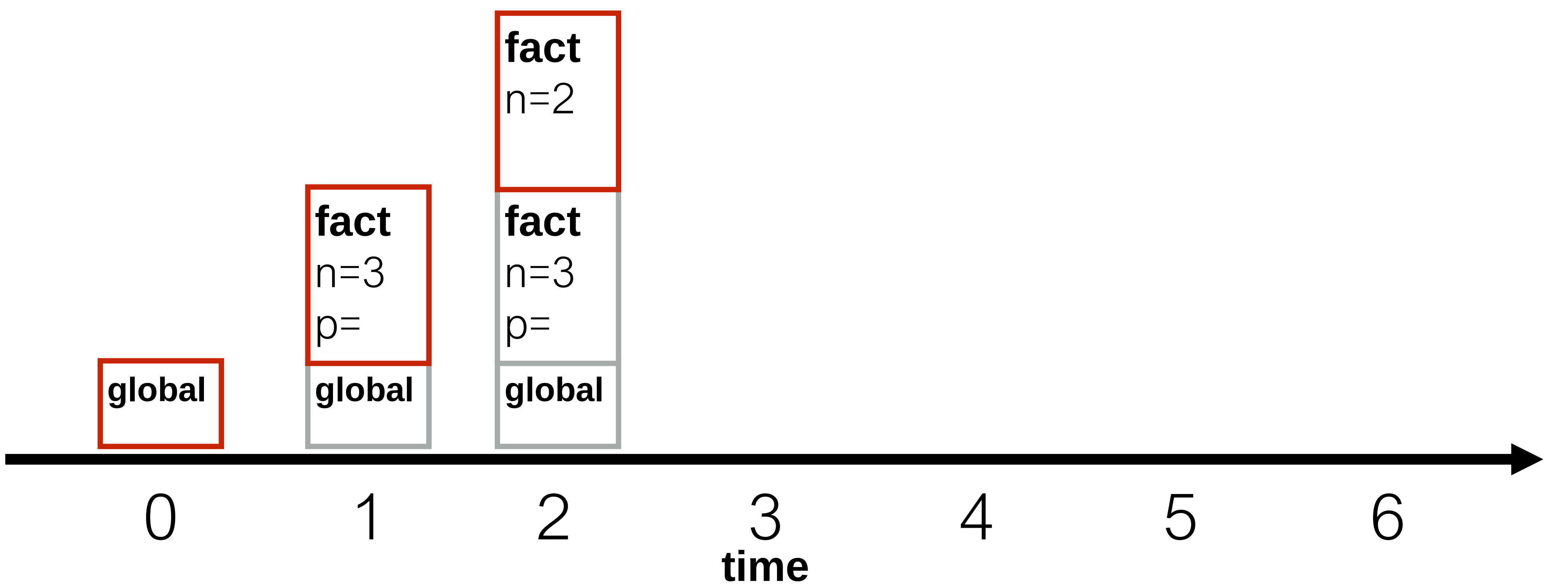

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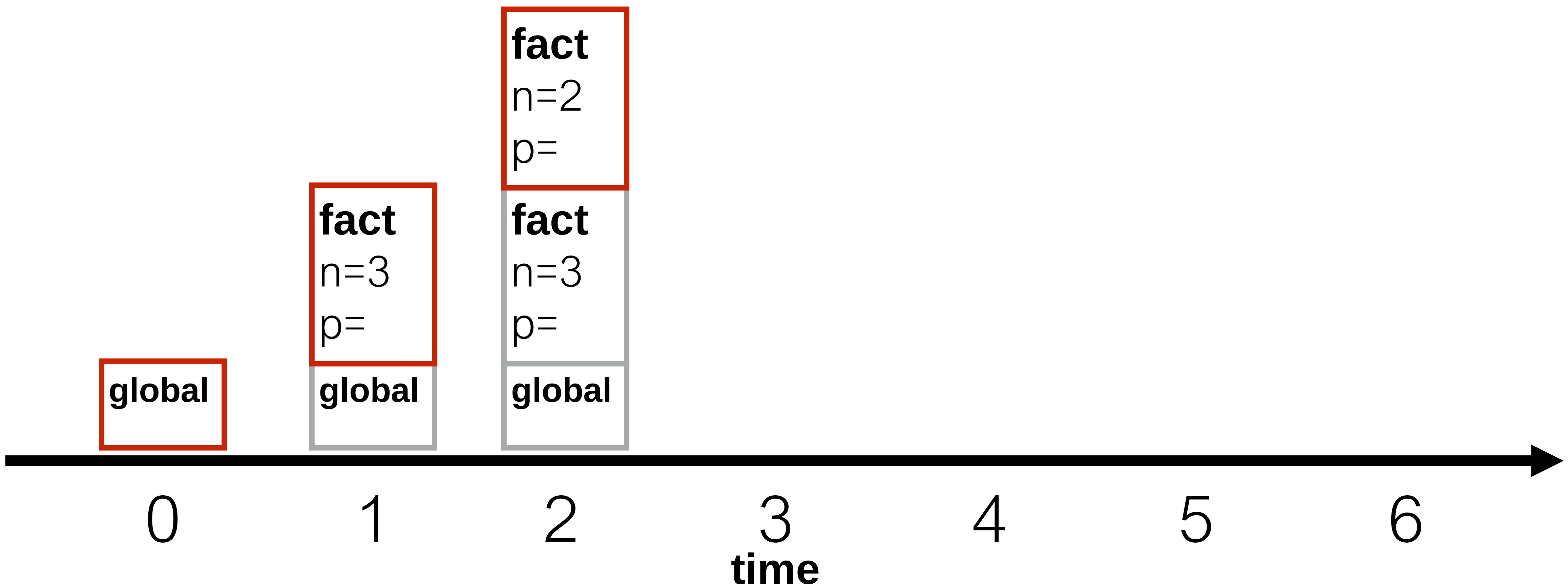

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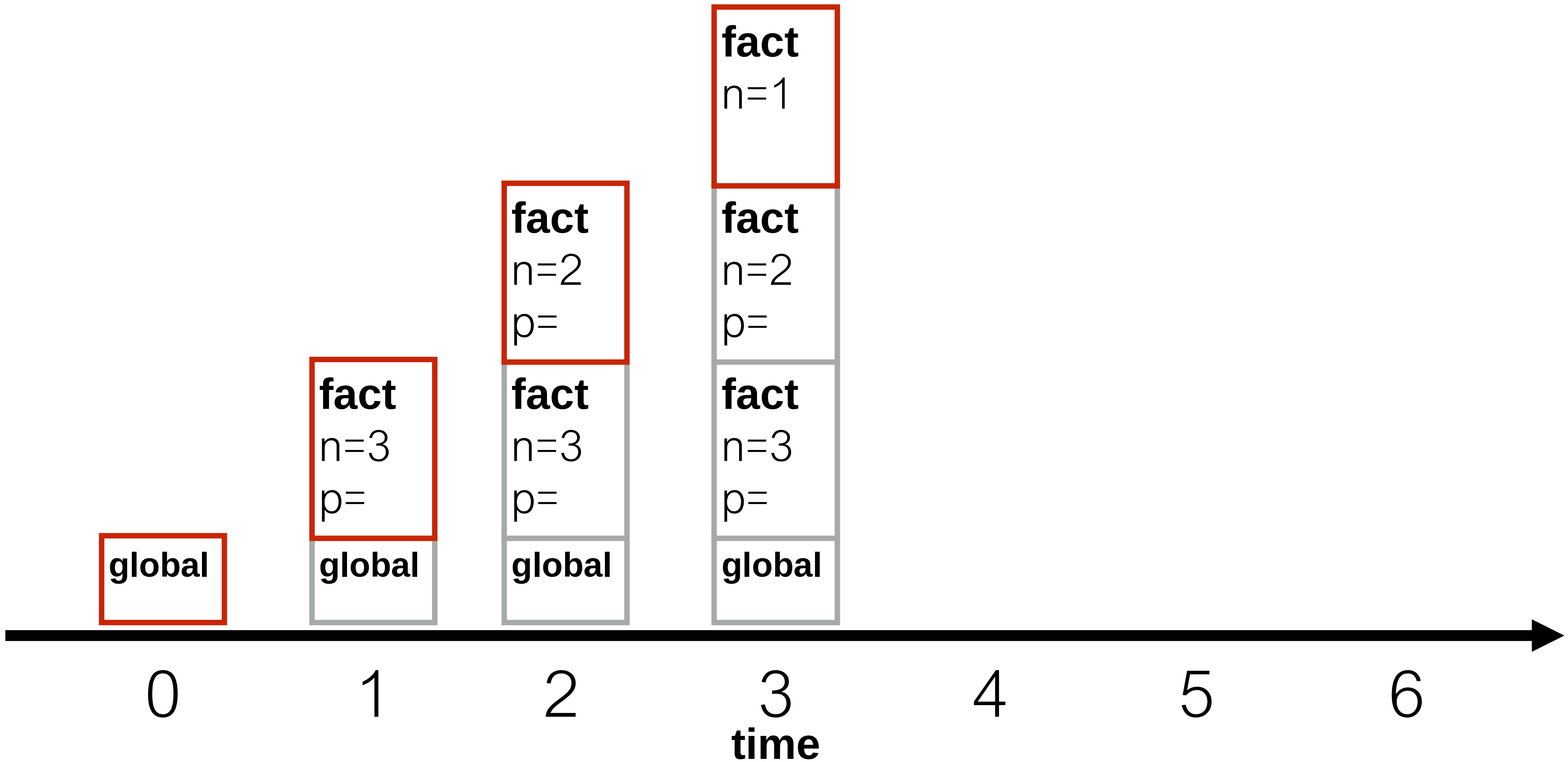

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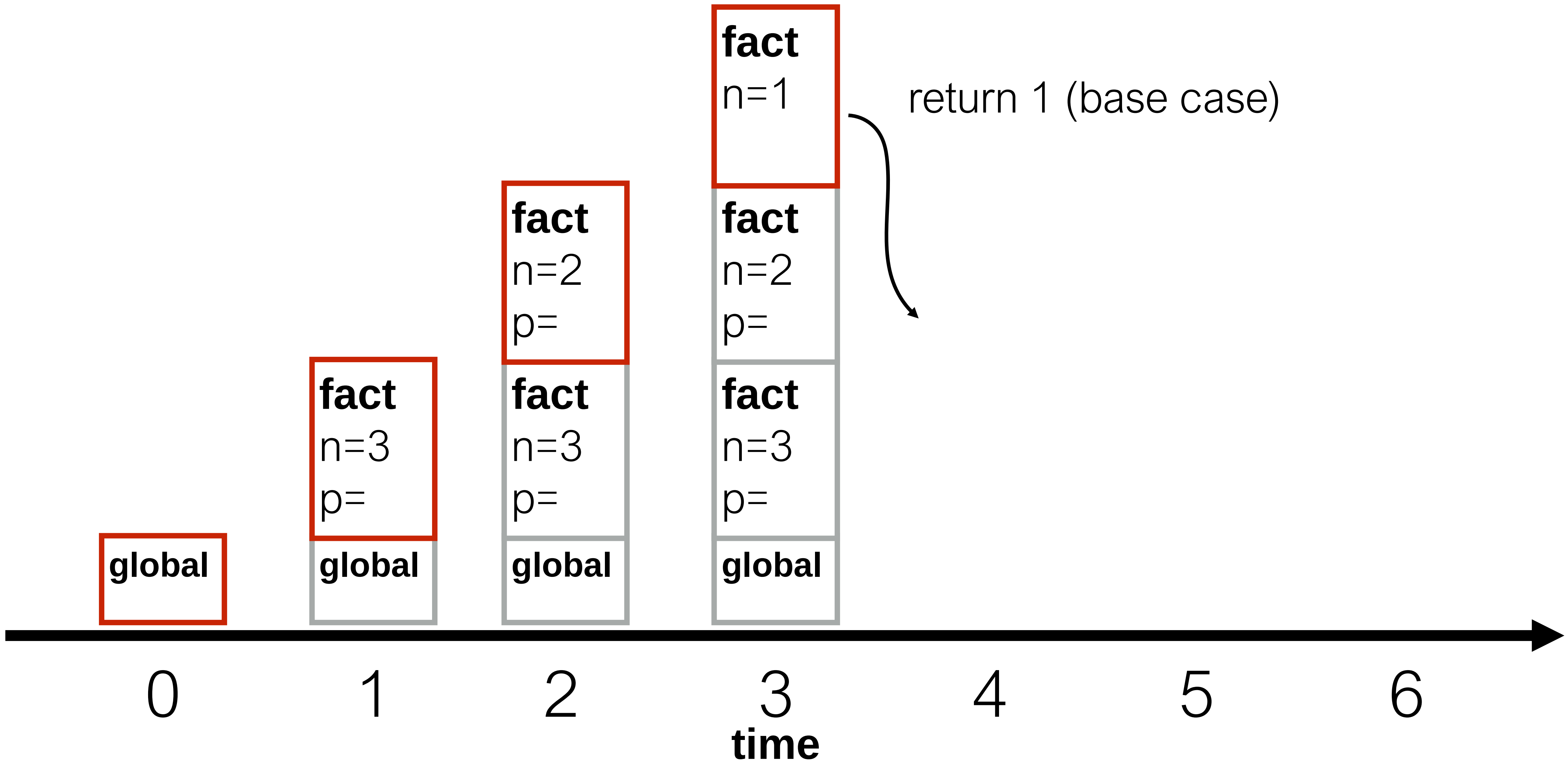

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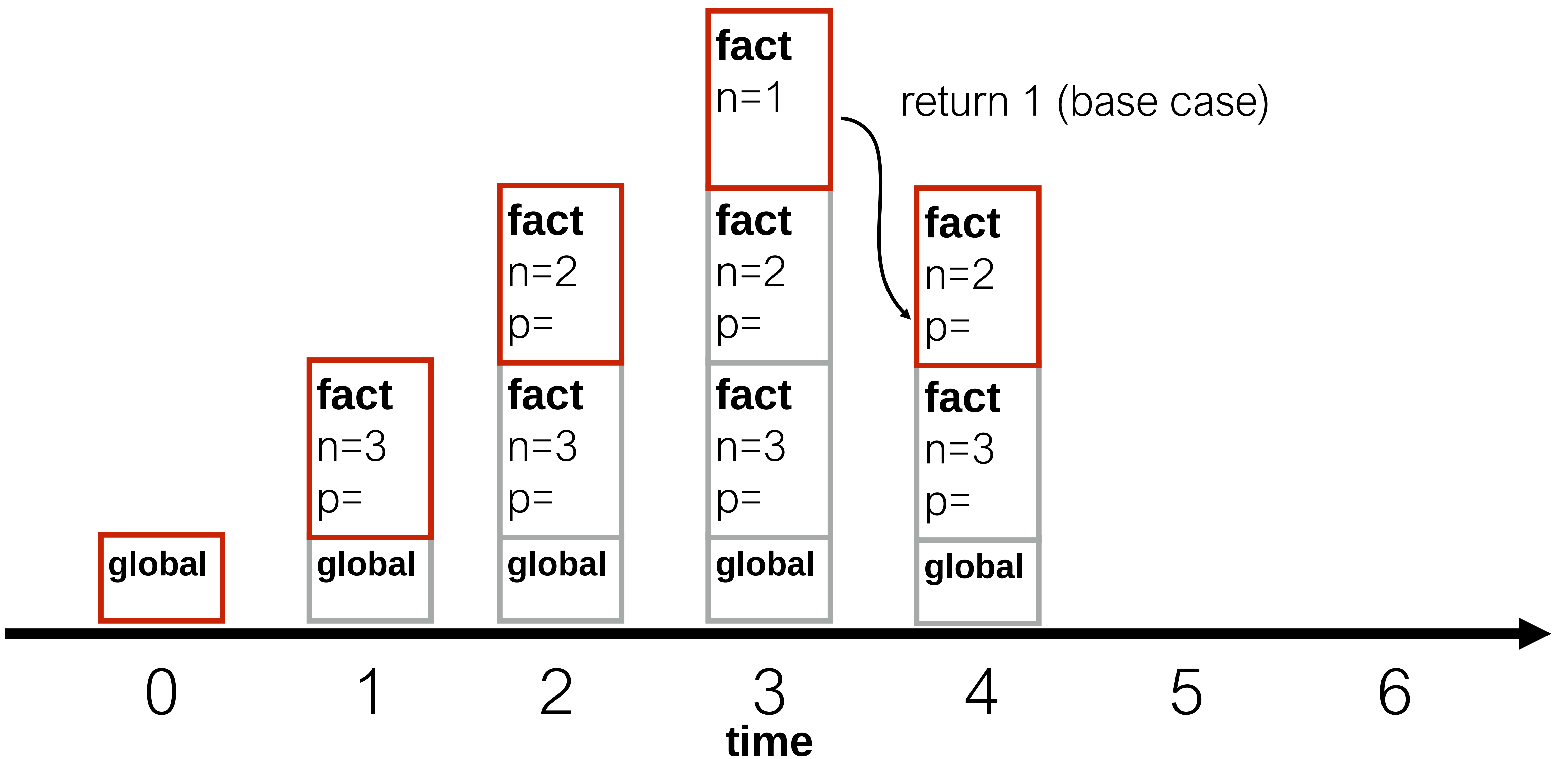

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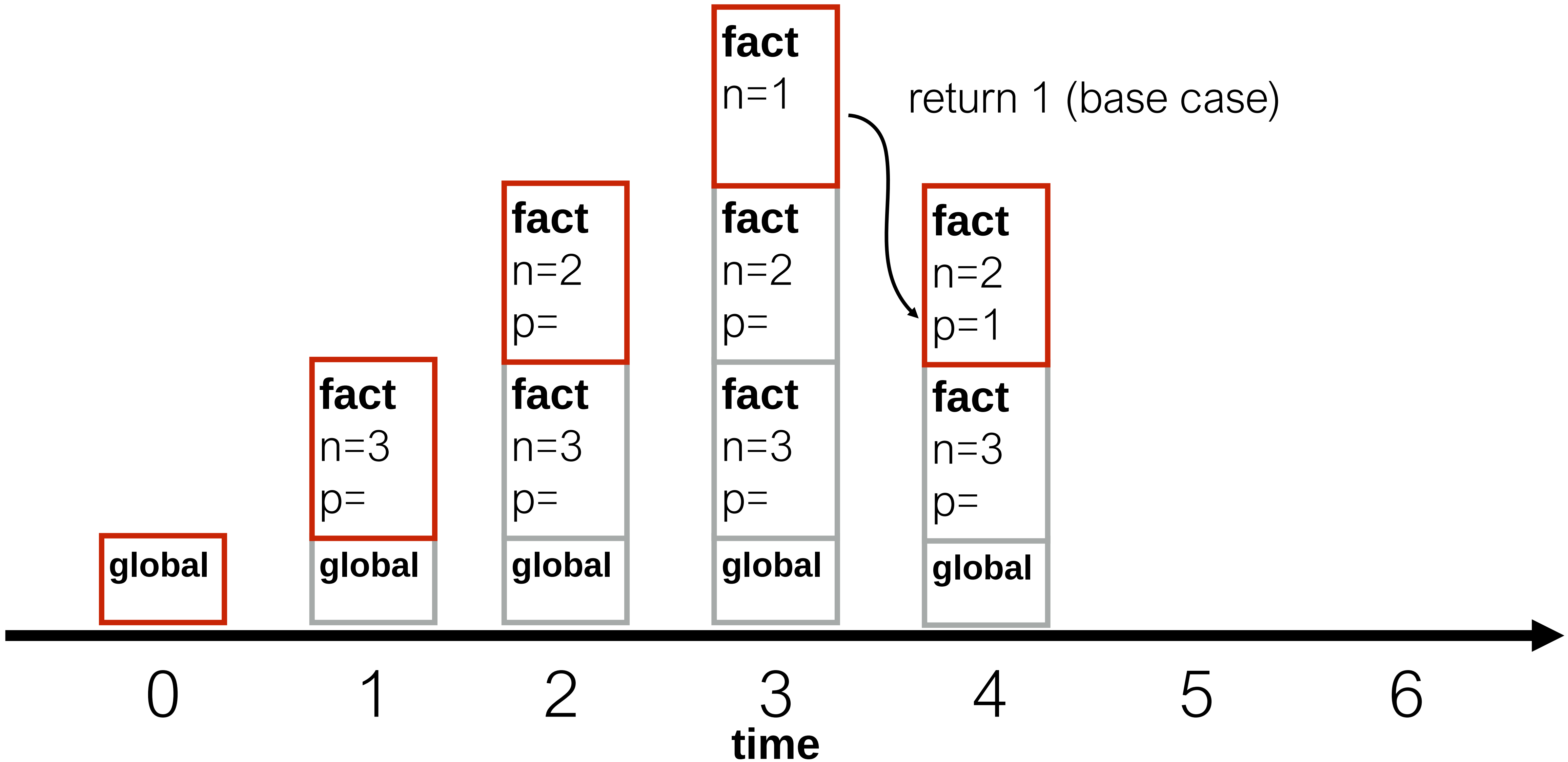

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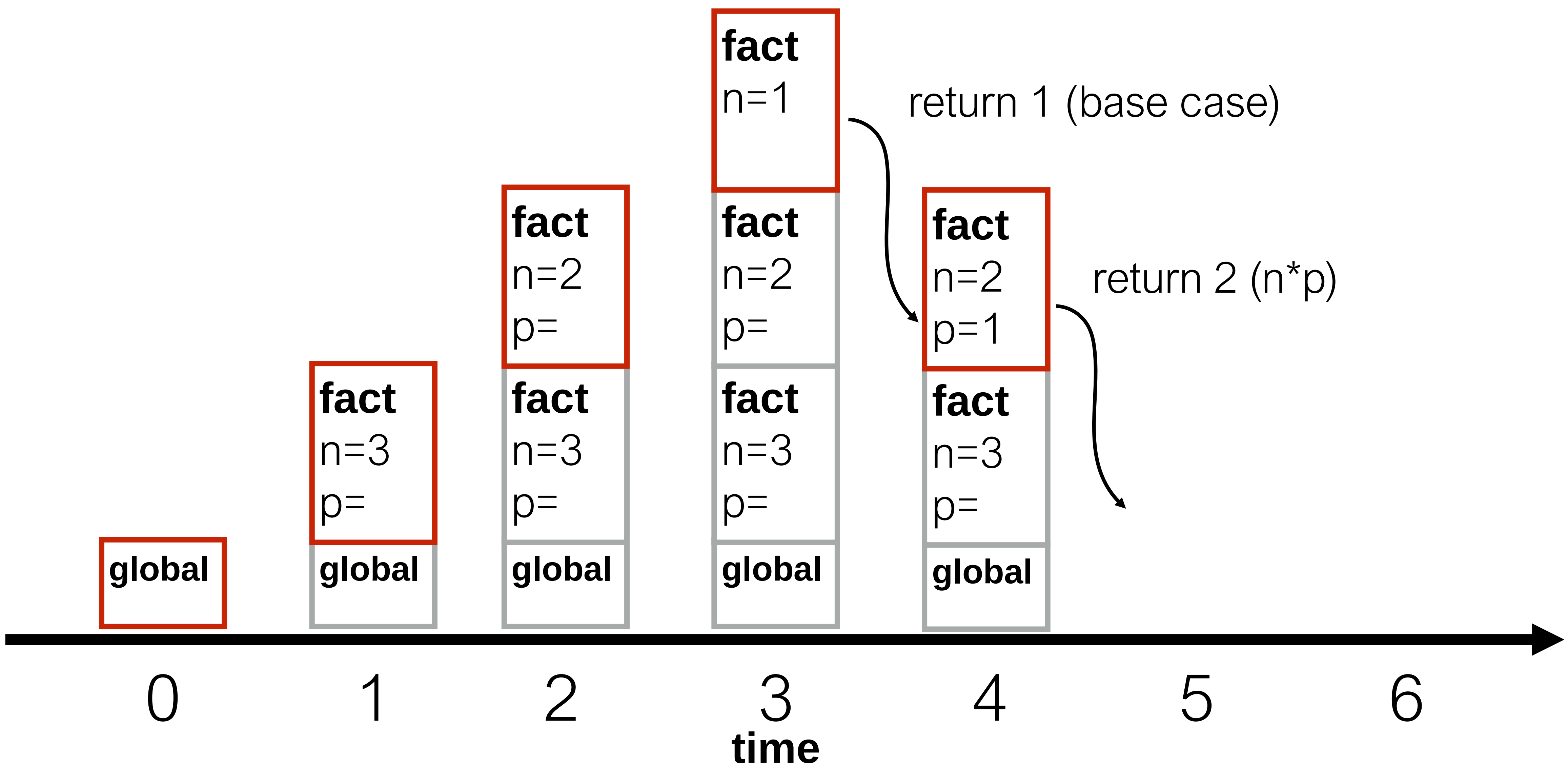

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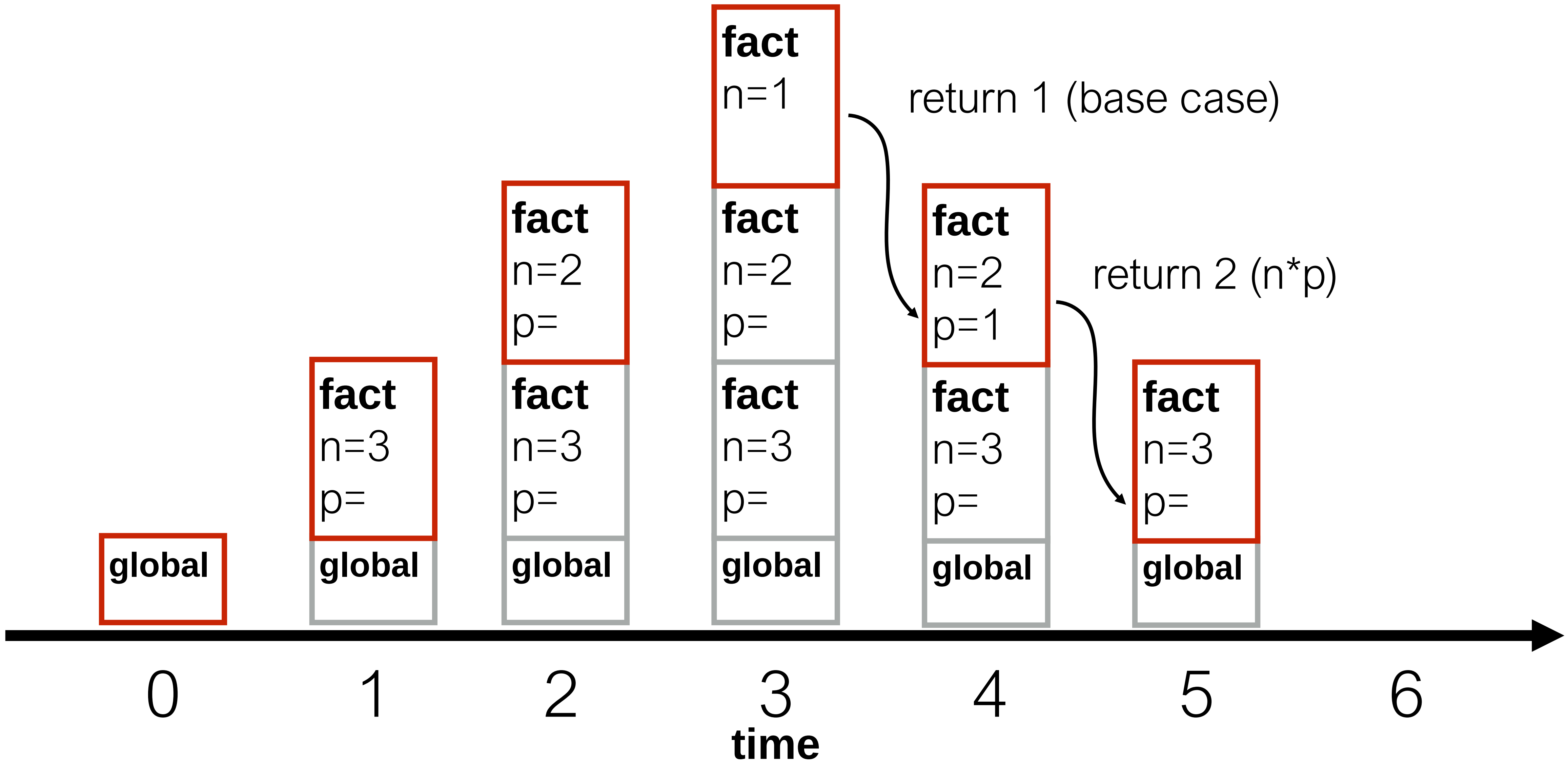

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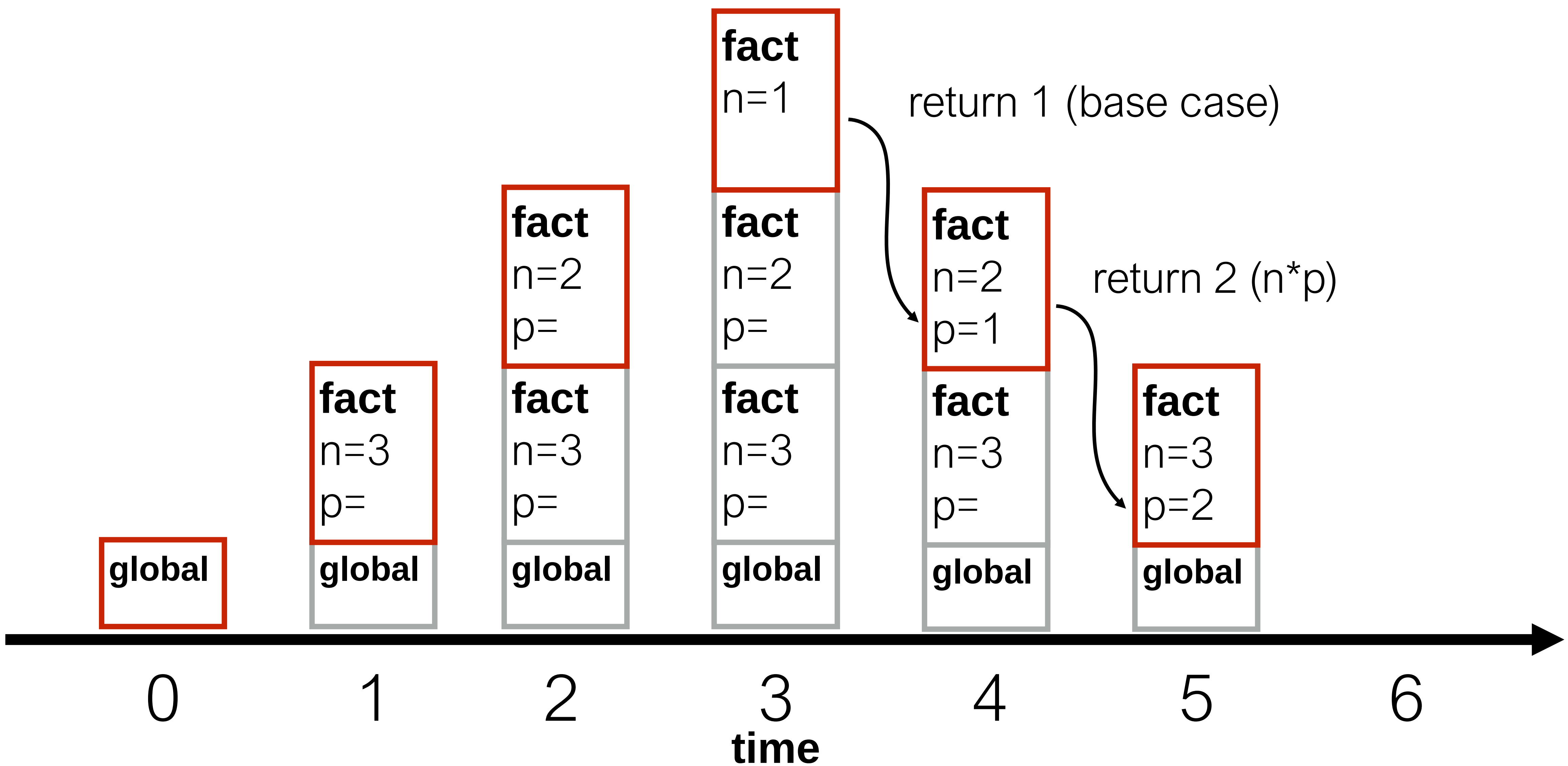

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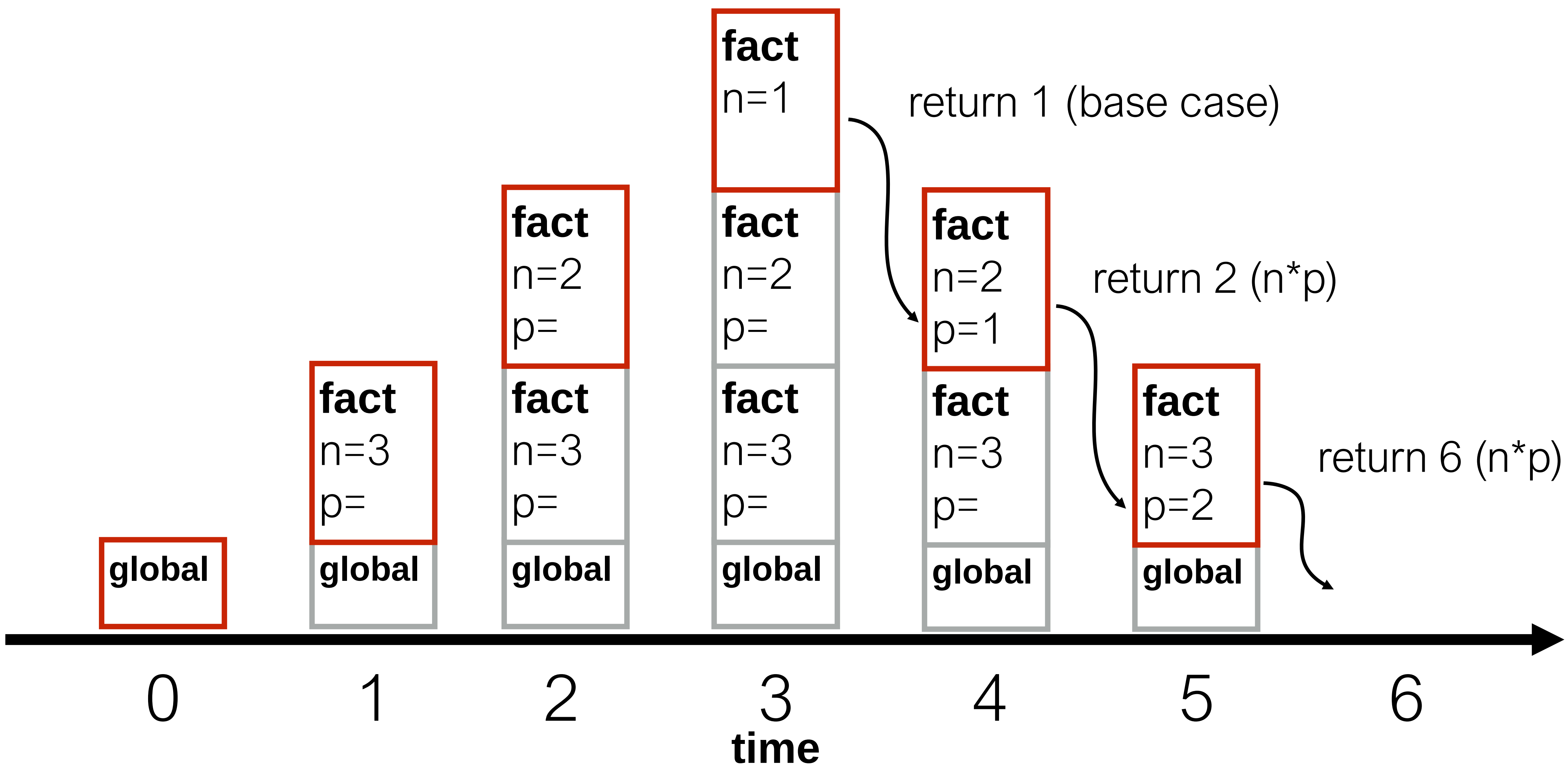

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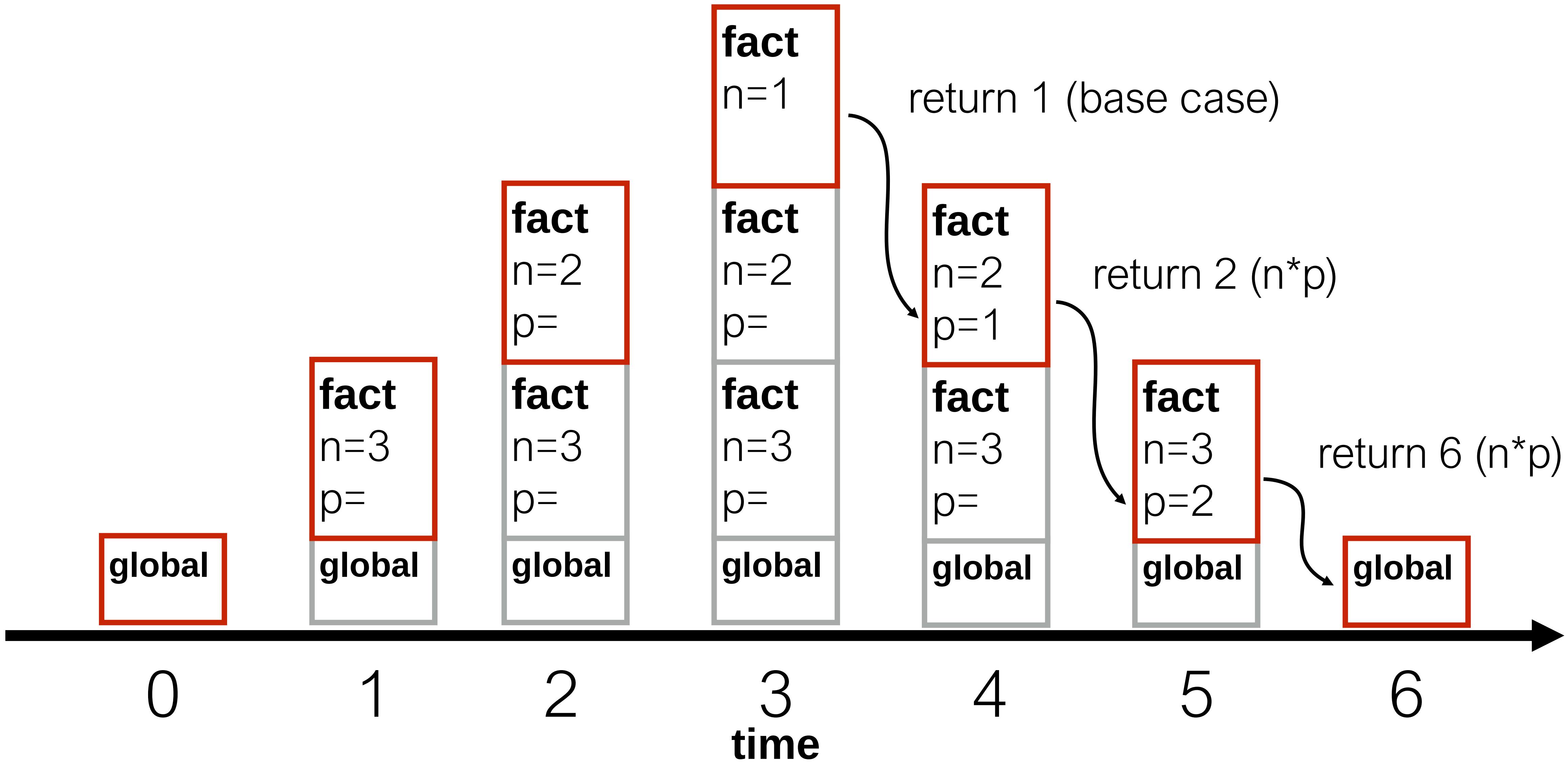

“Infinite” Recursion Bugs

What happens if:

-
-

```
def fact(n):
    if n == 1:
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    p =fact(n-1)
    return n * p
```

“Infinite” Recursion Bugs

What happens if:

- factorial is called with a negative number?
-

-1

```
def fact(n):
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    p = fact(n-1)
    return n * p
```

never terminates

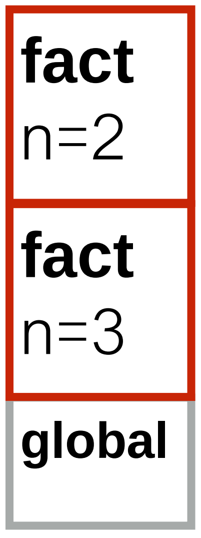
“Infinite” Recursion Bugs

What happens if:

- factorial is called with a negative number?
- we forgot the “n == 1” check?

3

def fact(n):
    ~~if n == 1:~~
        ~~return 1~~
    p = fact(n-1)
    return n * p



“Infinite” Recursion Bugs

What happens if:

- factorial is called with a negative number?
- we forgot the “n == 1” check?

3

def **fact**(n):
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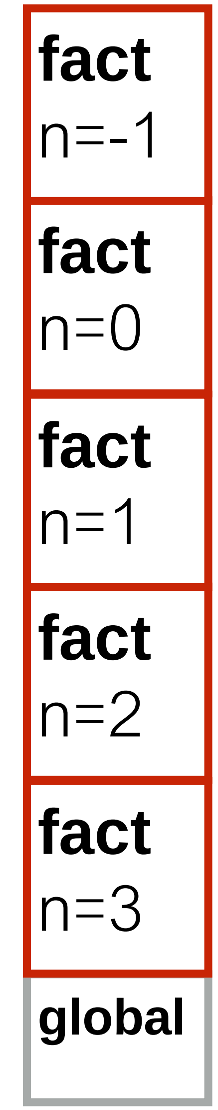
        ~~return 1~~
    p = **fact**(n-1)
    return n * p



“Infinite” Recursion Bugs

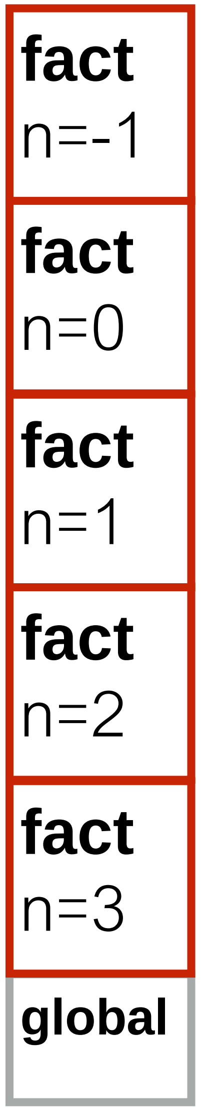
What happens if:

- factorial is called with a negative number?
- we forgot the “n == 1” check?

3

def **fact**(n):
~~if n == 1:~~
~~return 1~~
p = **fact**(n-1)
return n * p

never terminates



Coding Demos

Demo 1: Pretty Print

Goal: format nested lists of bullet points

Input:

- The recursive lists

Output:

- Appropriately-tabbed items

Example:

```
>>> pretty_print([“A”, [“1”, “2”, “3”,],
                  “B”, [“4”, [“i”, “ii”]]])
*A
  *1
  *2
  *3
*B
  *4
    *i
    *ii
```

Demo 2: Recursive List Search

Goal: does a given number exist in a recursive structure?

Input:

- A number
- A list of numbers and lists (which contain other numbers and lists)

Output:

- True if there’s a list containing the number, else False

Example:

```
>>> contains(3, [1,2,[4,[[3],[8,9]],5,6]])
True
>>> contains(12, [1,2,[4,[[3],[8,9]],5,6]])
False
```

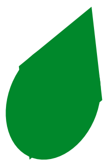
Conclusion: Review Learning Objectives

Learning Objectives: Recursive Information

What is a recursive definition/structure?

- Definition contains term
- Structure refers to others of same type
- Example: a dictionary contains dictionaries (which may contain...)

recursive case



base case

Learning Objectives: Recursive Code

What is recursive code?

- Function that sometimes itself (maybe indirectly)

Why write recursive code?

- Real-world data/structures are recursive; intuitive for code to reflect data

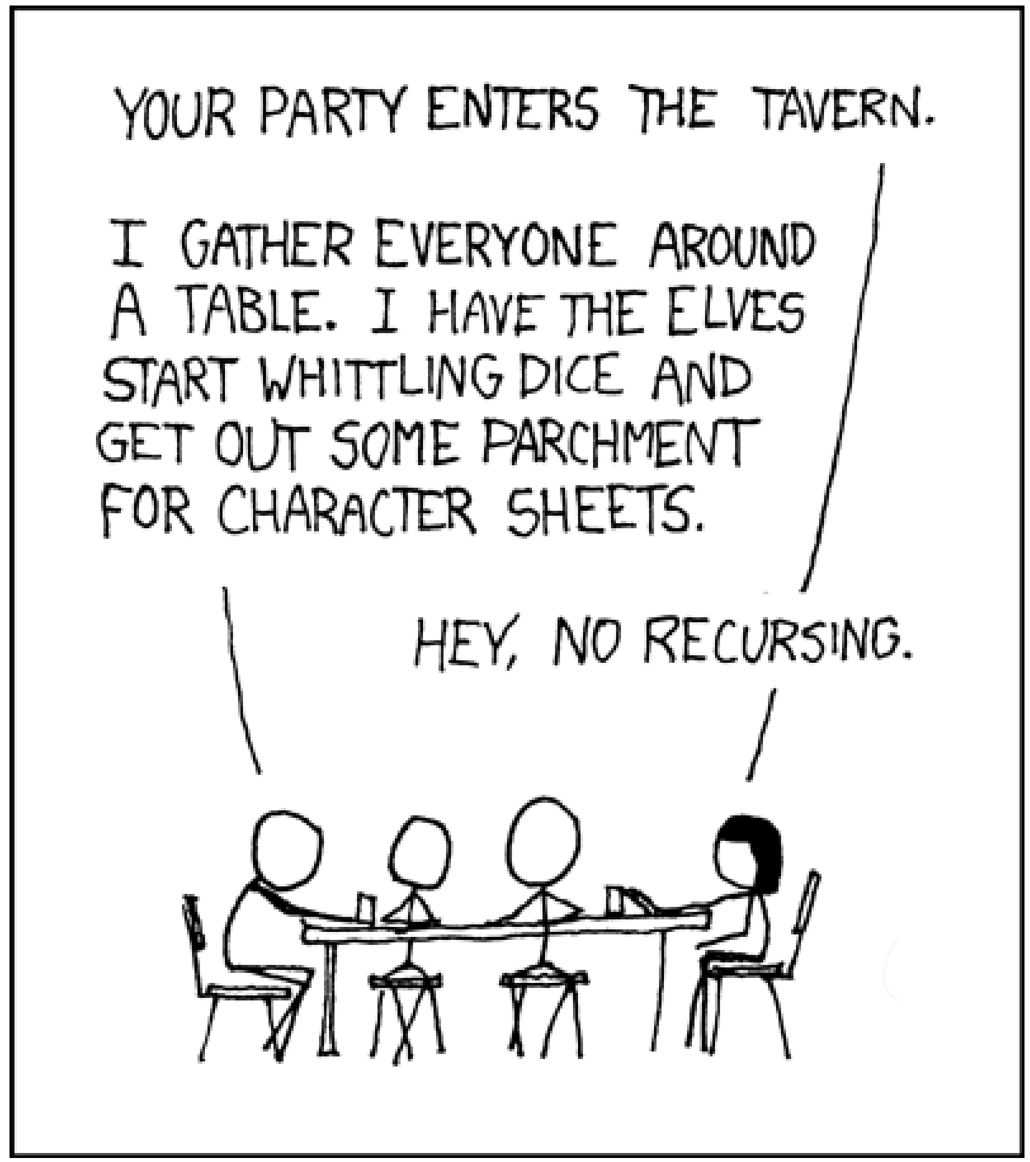
Where do computers keep local variables for recursive calls?

- In a section of memory called a “frame”
- Only one function is **active** at a time, so keep frames in a stack

What happens to programs with infinite recursion?

- Calls keep pushing more frames
- Exhaust memory, throw **StackOverflowError**

Questions?



https://xkcd.com/244/